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AN INVESTIGATION OF AIRCRAFT HEATERS

XXIV - THE HEAT METER IN THE TRANSIENT STATE

FOR UNIDIRECTIONAL HEAT TRANSFER

By L. M. K. Boelter, H. F. Poppendiek,
R. V. Dunkle, and J. T. Gier
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AN INVESTIGATION OF AIRCRAFT HEATERS

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SUMMARY

This report describes the behavior of the heat meter when it is used in thermal circuits which are under the influence of transient potentials. The transient heat transfer systems considered are as follows:

I. A heat meter suddenly placed upon a hot surface

II. A heat meter mounted upon the interior surface of the cabin wall of an uninsulated airplane, which is climbing through air of decreasing temperature

III. A heat meter mounted upon the interior surface of the composite cabin wall of an insulated airplane, which is climbing through air of decreasing temperature

Analytical and graphical solutions for cases I and II are presented. The analytical solutions were derived for idealized systems composed of lumped resistances and capacitances; the graphical solutions were effected by the Schmidt method (appendix A). In general, good agreement was obtained between the analytical and graphical solutions.

The solution of the differential equations which accurately describes a composite thermal system of distributed resistances and capacitances is difficult to obtain; thus, for case III only the graphical solution is presented.

This report is intended to show how analytical and graphical solutions for the transient state can be used to interpret the results obtained when heat meter readings are taken during the climb of an airplane. The solutions presented may not be directly applicable to a particular problem; however, the techniques used should serve as a guide for analyzing similar problems.

INTRODUCTION

In previous reports (references 1 and 2) the heat meter has been described and steady state added circuit resistance and temperature corrections are presented therein. This report contains an analysis of the heat meter when it is used in transient heat transfer systems; the effect of added circuit resistance and heat meter temperature corrections will not be considered for the transient state. In some thermal systems the temperature potentials are functions of time. Knowledge regarding the manner in which the heat meter responds to transients is therefore necessary. Analytical and graphical solutions for the transient state can be used to interpret the results obtained when heat meter readings are taken during the climb of an airplane. The solution of the differential equation which accurately describes a composite thermal system of distributed resistances and capacitances* is difficult to obtain. An idealized thermal system consisting of lumped resistances and capacitances* readily yields to description through a soluble differential equation. The distributed resistances should be small compared to the thermal resistance of the remainder of the circuit in order to obtain good approximations with this method. In the case of a heat meter mounted upon a wall, the method consists of treating the wall as a lumped capacitor located at the center of the wall instead of a distributed capacitor. The thermal resistance of the wall is considered to be divided equally on both sides of this central capacitor. Similarly the capacitance of the heat meter is lumped as a single capacitor at the center of the meter, and the thermal resistance of the heat meter is split equally to both sides of this capacitor. The Schmidt graphical method of solving a transient system should be used when the system contains a relatively large amount of distributed resistance and capacitance or when the temperature potentials or some of the circuit resistances

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*Distributed and lumped resistances and capacitances are discussed in appendix C.

vary in some irregular manner with time; as under these conditions the differential equation describing the system becomes unwieldy.

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SYMBOLS

A area through which the heat is being transferred, ft^2

a thermal diffusivity, $\frac{ft^2}{hr}$

C_1 1/2 thermal capacitance of heat meter, $\frac{Btu}{^oF}$

C_m thermal capacitance of heat meter, $\frac{Btu}{^oF}$

C_w thermal capacitance of wall, $\frac{Btu}{^oF}$

c_p heat capacity, $\frac{Btu}{lb\ ^oF}$

e base of natural logarithms

f_c unit thermal convective conductance, $\frac{Btu}{hr\ ft^2\ ^oF}$

k thermal conductivity, $\frac{Btu}{hr\ ft^2\ (^oF/ft)}$

L a significant dimension in equation (8), ft

n_1, n_2, n_3, n_4 constants obtained by simplifying equations (see appendix B)

q_1 heat flow from airplane interior into heat meter, $\frac{Btu}{hr}$

q_2 heat flow from heat meter into airplane wall, $\frac{Btu}{hr}$

q_3 heat flow from airplane wall to outside air, $\frac{Btu}{hr}$

q_M rate of heat transfer through heat meter, $\frac{Btu}{hr}$

$q_{M\infty}$ rate of heat transfer through heat meter at steady state, $\frac{Btu}{hr}$

R_1 1/2 heat meter resistance plus heat meter-air interface, $\frac{^oF\ hr}{Btu}$

R_3 air gap resistance plus 1/3 heat meter resistance, $\frac{^oF\ hr}{Btu}$

R_4 1/3 heat meter resistance, $\frac{^oF\ hr}{Btu}$

R_5 1/3 heat meter resistance plus heat meter-air interface resistance, $\frac{^oF\ hr}{Btu}$

R_6 1/2 heat meter resistance plus contact resistance plus 1/2 wall resistance, $\frac{^oF\ hr}{Btu}$

R_7 1/2 wall resistance plus outside air-wall interface resistance, $\frac{^oF\ hr}{Btu}$

T absolute temperature of air, ^oR

t_1, t_2 the temperatures of the hot and cold junctions of the heat meter thermopile, ^oF

t_m heat meter temperature at midsection, ^oF

t_w surface temperature, ^oF

u_m true airplane speed, ft/sec

α defined by equation $\tau_2 = \tau_c - \alpha\theta$, $^oF/hr$

γ weight density, lb/ft^3

$\Delta\theta$ time increment between steps in graphical method, hr

Δx distance increment, ft

θ time, hr

τ_1 ambient air temperature, $^{\circ}F$

τ_a air temperature outside plane, $^{\circ}F$

τ_c air temperature inside cabin (constant), $^{\circ}F$

f_r equivalent unit thermal conductance for radiation, $\frac{Btu}{hr\ ft^2\ ^{\circ}F}$

f unit thermal conductance due to radiation and convection, $\frac{Btu}{hr\ ft^2\ ^{\circ}F}$

R_a air gap resistance plus 1/2 heat meter resistance, $\frac{^{\circ}F\ hr}{Btu}$

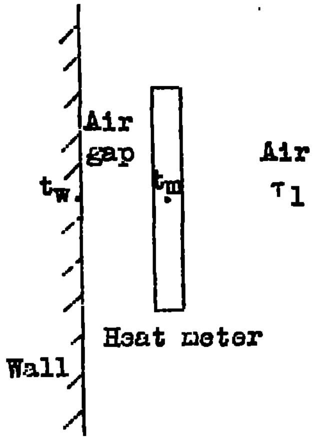
THERMAL SYSTEMS AND SOLUTIONS

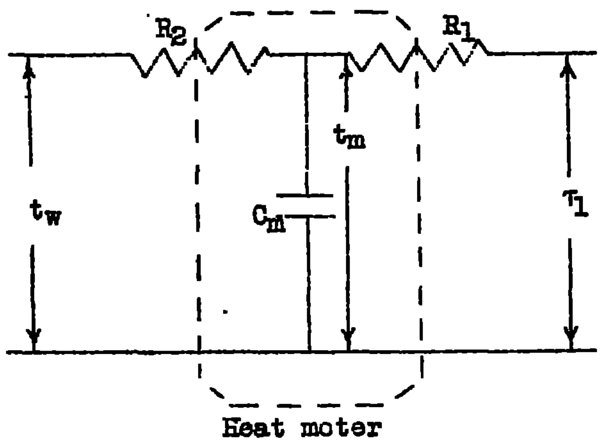
I. A Heat Meter Suddenly Placed upon a Hot Surface

A heat meter initially at air temperature, τ_1, is suddenly placed against a hot surface. The time-temperature history of the heat meter under these conditions is desired. The thermal system consists of a hot surface, an air gap (contact resistance), the heat meter, and a unit thermal conductance due to convection and radiation (heat meter-air interface resistance). Experimental data were taken for the above conditions. Initially the temperatures of the air, heat meter, and air gap were at 78° F, while the temperature of the hot surface was at 122.8° F. A thermocouple located under the first lamination of the heat meter was used to obtain a time-temperature history of the heat meter (shown in fig. 1). The temperature of the hot surface increased slightly after the heat meter was placed upon the surface. At steady state the unit thermal conductance $(f_c + f_r)$ was experimentally determined to be 2 Btu/hr ft^2 °F. The properties of the elements in the thermal system are as indicated in table I. The product γc_p for air is small and can be considered to be zero in the air gap for this system.

A. Analytical determination of the time-temperature history of a heat meter suddenly placed on a hot surface.

The System





C_m thermal capacitance of heat meter, $\frac{\text{Btu}}{{}^\circ\text{F}}$

R_1 1/2 heat meter resistance plus surface resistance, $\frac{{}^\circ\text{F hr}}{\text{Btu}}$

R_2 air gap resistance plus 1/2 heat meter resistance, $\frac{{}^\circ\text{F hr}}{\text{Btu}}$

t_m average heat meter temperature, $^\circ$F

t_w hot surface temperature, $^\circ$F

θ time, hr

τ_1 ambient air temperature, $^\circ$F

Equating the heat flow from the wall to the meter to the rate of heat storage in the meter plus the heat flow from the meter surface to the surroundings (postulating unidirectional flow; i.e., heat flow from edges = 0)

$$\frac{t_w - t_m}{R_2} = C_m \frac{dt_m}{d\theta} + \frac{t_m - \tau_1}{R_1} \qquad (1)$$

or

$$C_m \frac{dt_m}{d\theta} + \left(\frac{R_1 + R_2}{R_1 R_2}\right) t_m = \frac{t_w}{R_2} + \frac{\tau_1}{R_1} \tag{2}$$

or

$$\frac{dt_m}{d\theta} + P t_m = Q \tag{3}$$

where

$$P = \frac{1}{C_m}\left(\frac{R_1 + R_2}{R_1 R_2}\right), \qquad Q = \frac{1}{C_m}\left(\frac{t_w}{R_2} + \frac{\tau_1}{R_1}\right).$$

The solution of the differential equation (3) is (see reference 9):

$$t_m = \frac{Q}{P} + \left(1 - \frac{Q}{P}\right) e^{-P\theta} \tag{4}$$

Substituting for P and Q

$$t_m = \frac{\left(\frac{t_w}{R_2} + \frac{\tau_1}{R_1}\right)}{\left(\frac{R_1 + R_2}{R_1\ R_2}\right)} + \left[\tau_1 - \frac{\left(\frac{t_w}{R_2} + \frac{\tau_1}{R_1}\right)}{\left(\frac{R_1 + R_2}{R_1\ R_2}\right)}\right] e^{-\left(\frac{R_1 + R_2}{R_1\ R_2}\right)\frac{\theta}{C_m}}$$

or

$$\frac{t_m - \tau_1}{t_w - \tau_1} = \frac{R_1}{R_1 + R_2}\left(1 - e^{-\left(\frac{R_1 + R_2}{R_1\ R_2}\right)\frac{\theta}{C_m}}\right) \tag{5}$$

where $\left(\frac{R_1\ R_2\ C_m}{R_1 + R_2}\right)$ is a time constant which is a function of the heat meter resistance and capacitance and the air gap and heat meter-air interface resistance.

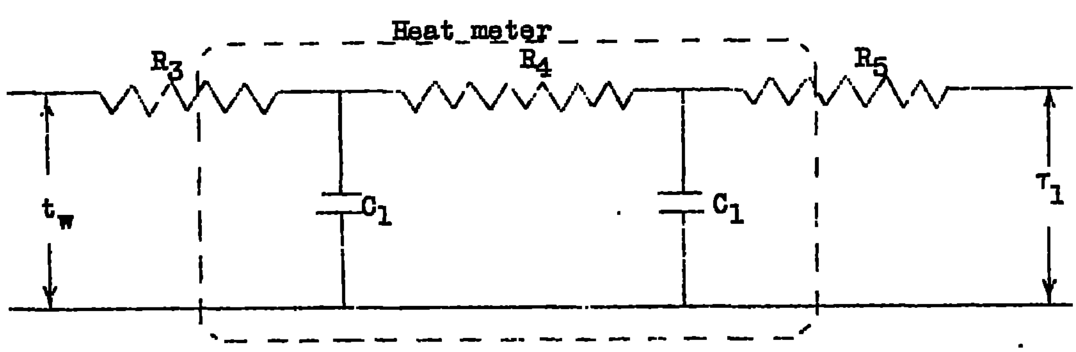
This equation represents the temperature-time history for a heat meter suddenly placed on a hot surface of constant temperature. The results are tabulated in table II and plotted together with experimental points in figure 1. The discrepancy between the experimental and calculated points is mainly due to the fact that the hot plate temperature . . .

increased slightly due to the increased thermal resistance added by the heat meter.

B. Graphical determination of the time-temperature history of a heat meter suddenly placed upon a hot surface. Figure 2 exhibits the Schmidt solution of the time-temperature history of a heat meter suddenly placed upon a hot surface. An explanation of the Schmidt technique for solving transient heat transfer problems is given in appendix A. The time-temperature history at a point inside the heat meter where the thermocouple is located is taken from the Schmidt solution, figure 2, tabulated in table III, and plotted with the experimental data in figure 1. The deviation between the experimental and Schmidt time-temperature histories is seen to be negligible. Experimentally it took 2.1 minutes for the temperature of the heat meter to reach 95 percent of the steady state heat meter temperature. The time required for the temperature of the heat meter to reach 95 percent of the steady state heat meter temperature as determined by the two methods is tabulated below.

Experimental result (min)	Analytical result (min)	Graphical result (min)
2.1	1.4	2.0

C. Analytical determination of the time-heat flow history of a heat meter suddenly placed on a hot surface. When the capacitance of the heat meter is lumped into two capacitors the following equation of heat flow through the heat meter as a function of time results:



The ratio of the heat flow through the heat meter at any time θ to the steady state heat flow (θ = infinity) is given by

$$\frac{q_M}{q_{M_\infty}} = 1 + \left[\frac{n_2}{n_1 - n_2} + \frac{R_3 + R_4 + R_5}{(n_1 - n_2)\; C_1\; R_3\; R_4}\right] e^{n_1 \theta}$$

$$+ \left[\frac{n_2}{n_1 - n_2} + \frac{R_3 + R_4 + R_5}{(n_2 - n_1)\; C_1\; R_3\; R_4}\right] e^{n_2 \theta} \qquad (6)$$

The method used in solving this equation and the definition of the terms n_1 and n_2 are given in appendix B.

When the values n_1, n_2, R_3, R_4, R_5, C_1, and θ are evaluated for this particular case, the following results:

$$\frac{q_M}{q_{M_\infty}} = 1 + 4.13\ e^{-153\,\theta} - 5.13\ e^{-2250\,\theta} \qquad (7)$$

Table IV and figure 3 exhibit the time-heat flow history of the heat meter as given by equation (7).

D. Graphical determination of the time-heat flow history of a heat meter suddenly placed on a hot surface.- Because the Schmidt method yields the temperature throughout the heat meter as a function of time, the temperature difference across a fixed resistance in the heat meter (as measured by the heat meter thermopile) is known as a function of time. The heat flow-time history of the heat meter calculated from the drop in temperature across the fixed resistance as obtained by the Schmidt method is tabulated in table V and plotted in figure 3. The Schmidt method reveals that the heat flow through the heat meter is within 5 percent of the steady state heat transfer rate after 1.4 minutes, while the analytical solution yields 1.7 minutes.

II. Transient Behavior of a Heat Meter Mounted upon the Inside of an Uninsulated Cabin Wall of an Airplane in Flight

An airplane leaves the ground, flying at the rate of 146 feet per second with respect to the still air, and at an angle of 20° with the horizontal. Thus the vertical rate of climb is 50 feet per second. The airplane climbs at this rate for $9\frac{1}{4}$ minutes, at which time an elevation of 27,750 feet has been reached. The temperature of the still air on the ground is 60° F, while at an elevation of 27,750 feet it is -40° F. Some of the atmospheric data at different elevations are tabulated in table VI. (See reference 3.) How long must an airplane fly at 27,750 feet, after having attained that elevation, before the rate of heat transfer through the heat meter mounted on the inside of the cabin wall is within 5 percent of the steady state rate of heat transfer? As the airplane climbs, the air temperature, air density, and the outside unit thermal convective conductance, vary. The average unit thermal convective conductance between the air and the outside of the cabin wall may be expressed as (cf. reference 4):

$$f_c = 0.64\ T^{0.3}\left(\frac{u_m\ \gamma}{L^{0.25}}\right)^{0.8} \qquad (8)$$

The significant dimension chosen in this case is 6 feet. (The significant dimension L in equation (8) is the length of a flat plate over which the average unit thermal convective conductance is desired. An average unit thermal convective conductance over a 6-foot length was obtained.) By using the data tabulated in table VI (reference 3), the unit thermal convective conductance is calculated as a function of elevation and time of flight as shown in table VII. The properties of the elements of the thermal system are as indicated in table VIII. The unit thermal conductance due to convection and radiation on the inside of the cabin is postulated to be 1 Btu/hr ft² °F.

A. Analytical determination.

Symbols

B_3 integration constant

a constant defined in equation (B7)

B_4 integration constant

b constant defined in equation (B8)

c constant defined in equation (B9)

n_3, n_4 constants defined in equation (B11)

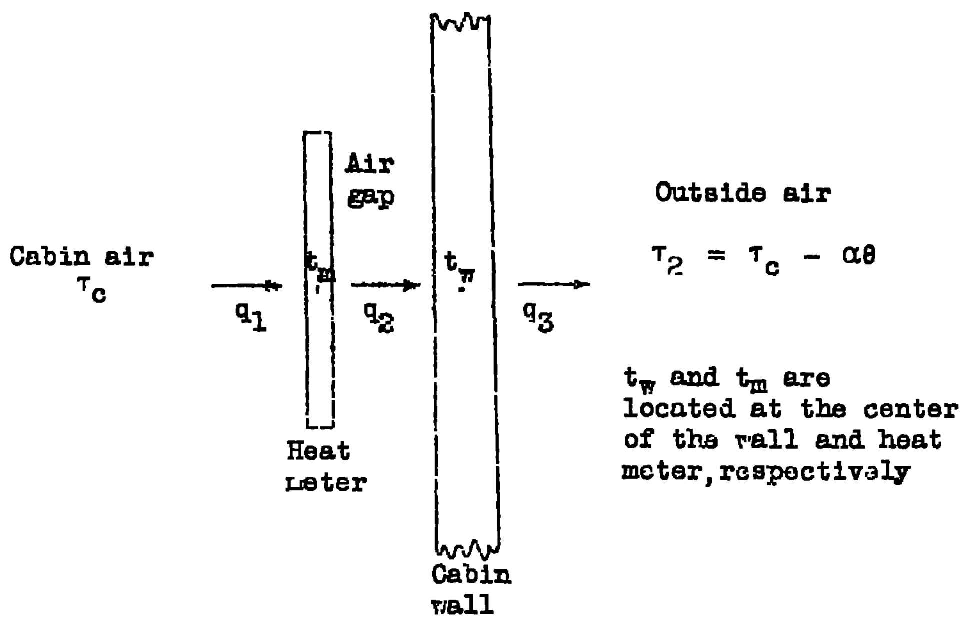
R_1 first lumped thermal resistance, inside surface resistance plus 1/2 heat meter resistance, $^{\circ}F/\frac{Btu}{hr}$

R_6 second lumped thermal resistance, 1/2 heat meter resistance plus contact resistance plus 1/2 wall resistance, $^{\circ}F/\frac{Btu}{hr}$

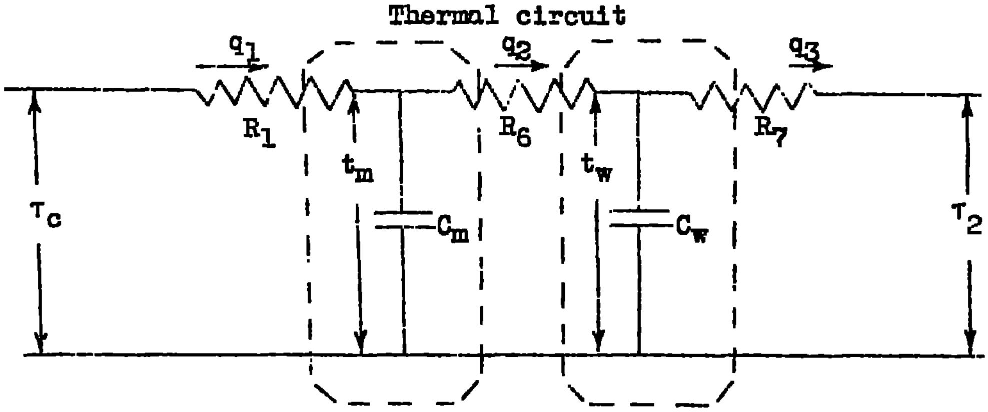
t_w average wall temperature, $^{\circ}F$

α coefficient of temperature drop with time, $\frac{^{\circ}F}{hr}$ = constant

The System



Thermal circuit



The outside air temperature, τ_2, varies linearly with time and may be expressed as $\tau_2 = \tau_c - \alpha\theta$. ($\alpha$ is a constant.)

The complete analytical solution is contained in appendix B. The differential equation obtained is:

$$C_m\, C_w\, \frac{d^2 t_m}{d\theta^2} + \left[C_m\left(\frac{R_6 + R_7}{R_6\, R_7}\right) + C_w\left(\frac{R_1 + R_6}{R_1\, R_6}\right)\right]\frac{dt_m}{d\theta}$$

$$+\left(\frac{R_1 + R_6 + R_7}{R_1\, R_6\, R_7}\right) t_m = \left(\frac{R_1 + R_6 + R_7}{R_1\, R_6\, R_7}\right)\tau_c - \frac{\alpha\theta}{R_6\, R_7} \qquad (9)$$

(This equation and equation (B6) in appendix B are identical.) The rate of heat transfer through the heat meter as a function of time is

$$q_M = \frac{\alpha\theta}{R_1 + R_6 + R_7} - \frac{b\,\alpha\, R_1\, R_6\, R_7}{(R_1 + R_6 + R_7)^2} + \frac{C_m\,\alpha\, R_1}{2(R_1 + R_6 + R_7)}$$

$$- \frac{\alpha\, e^{n_3\theta}}{(n_3 - n_4)(R_1 + R_6 + R_7)}\left[1 + \frac{b\, n_4\, R_1\, R_6\, R_7}{R_1 + R_6 + R_7} - \frac{C_m\, R_1\, n_1}{2}\right]$$

$$- \frac{\alpha\, e^{n_4\theta}}{(n_4 - n_3)(R_1 + R_6 + R_7)}\left[1 + \frac{b\, n_3\, R_1\, R_6\, R_7}{R_1 + R_6 + R_7} - \frac{C_m\, R_1\, n_3}{2}\right] \qquad (10)$$

(This equation and equation (B26) are identical.)

Tho physical meaning of this equation can be explained as follows:

The first term represents the heat flow if a steady state is reached at each point. The second and third terms together represent the lag in heat flow reading due to the capacities in the system, after the effect of the initial conditions has vanished. The last two terms represent the effect of the initial conditions. The whole equation represents the heat flow as read by the meter as a function of time.

The numerical evaluation of this equation is plotted in figure 4 and the calculated values are given in table IX. Three curves are present in figure 4. Curve I shows the heat flow through the wall <u>without</u> the heat meter if a steady state is reached at each elevation. Curve II gives the heat flow reading which would be indicated by the heat meter if a steady state were reached at each elevation. Curve III gives the heat flow as read by the meter under the assumed conditions. In figure 8 the calculated values of heat flow are plotted with the values obtained by the Schmidt method. The greatest difference between the two curves is due to the fact that an average thermal resistance between the outside air and the wall was employed for the analytical solution; whereas, for the graphical solution, the appropriate variable resistance was used.

The equations (9) and (10) apply to the climbing airplane, but the heat flow curve after the airplane levels off is also plotted. The differential equations and solution for this case are nearly the same as before, the difference being that τ_a is now constant. The initial conditions are chosen at the instant the plane levels off. The solution is:

$$q_M = q_{M\infty} + B_3 e^{n_3 \theta} + B_4 e^{n_4 \theta} \tag{11}$$

where B_3 and B_4 are determined by the heat flow conditions at the instant the plane levels off. When the appropriate values are substituted, the resultant equation is:

$$q_M = 87.0 - 4.13 e^{-435 \theta} + 28.83 e^{-55.5 \theta} \tag{12}$$

The calculated values of heat flow are given in table IX and are also plotted in figure 4 and represent the part of curve III for values of time over 0.155 hour. It is seen that after leveling off, it takes approximately 0.035 hour, or 2.1 minutes for the heat meter reading to reach 5 percent of the steady state value.

<u>B. Graphical determination</u>.- The Schmidt solution of the time-temperature history throughout a heat meter which was attached to the inside of the wall of an airplane in flight was constructed in the following manner. First, the thermal resistances of the circuit elements of the system were represented by a convenient scale. Next, the initial temperature distribution was indicated. Since the thermal resistance of the cabin wall was negligible, it was represented by a line on the Schmidt solution. The thermal capacitance of the air gap was small and neglected. Thus the temperature gradient for a <u>particular time increment</u> was constant across the air

gap. The heat meter was divided into one slab only as the temperature gradient was rather small. Since $\Delta x = 0.004$ foot and $a = 0.00264$ ft^2/hr, the time increment was $\frac{\Delta x^2}{2a} = \frac{0.004^2}{2(0.00264)} = 0.00303$ hour or 0.18 minute. The outside air temperature and the value of k/f_c are plotted against time in figure 6. Thus for any time (determined by the number of time increments) the directional point R can be determined. Table X and figure 8 exhibit the time-heat flow results through the heat meter obtained by the Schmidt method shown in figure 7. It would take 1.25 minutes for rate of heat transfer through the heat meter to reach 5 percent of the steady state value after the airplane leveled off.

The steady state rate of heat transfer is that rate which would occur at each particular elevation if the plane were to level off and fly at that elevation for an infinite time. From figure 8 it can be seen that the transient rate of heat transfer as measured by the heat meter is greater than the steady state rate of heat transfer. This observation can be explained as follows:

A greater amount of the heat stored in that section of the heat meter nearest the decreasing outside air will flow out of the heat meter than that stored in the section of the heat meter which is farthest from the outside air. Thus the temperature of the first section will be decreasing at a greater rate than that of the second section, and since the heat meter thermopile measures the temperature difference between these two sections within the heat meter, the transient rate of heat transfer as measured by the heat meter will be greater than the steady state value.

III. The Transient Behavior of a Heat Meter Mounted upon the Inside of an Insulated Cabin Wall of an Airplane in Flight

If a heat meter is attached to an insulated cabin wall of an airplane flying under the previously specified conditions (illustrative example II), how long must the airplane fly at 27,750 feet in order that the heat flow through the heat meter be within 5 percent of the steady state heat flow when the heat meter is mounted between the wall and insulation, and when the heat meter is mounted at the insulation-cabin air interface? The thermal circuit consists of an outside unit thermal conductance, a cabin wall, a heat meter, a contact resistance, an insulation material, and an inside unit thermal conductance as shown in figure 9. The distributed resistance and capacitance of the heat meter is negligible compared to the large distributed resistance and capacitance of the insulating material. Thus the heat meter very nearly measures a rate of heat transfer that would occur if the heat meter were not in the thermal circuit. Since the system contains a relatively large distributed resistance and capacitance of the insulating material, the analytical method of lumping the circuit constants becomes cumbersome, and the Schmidt method of solution is used. The properties of the insulating material are:

$$k = 0.025 \text{ Btu/hr ft}^2 \frac{^\circ F}{ft}$$

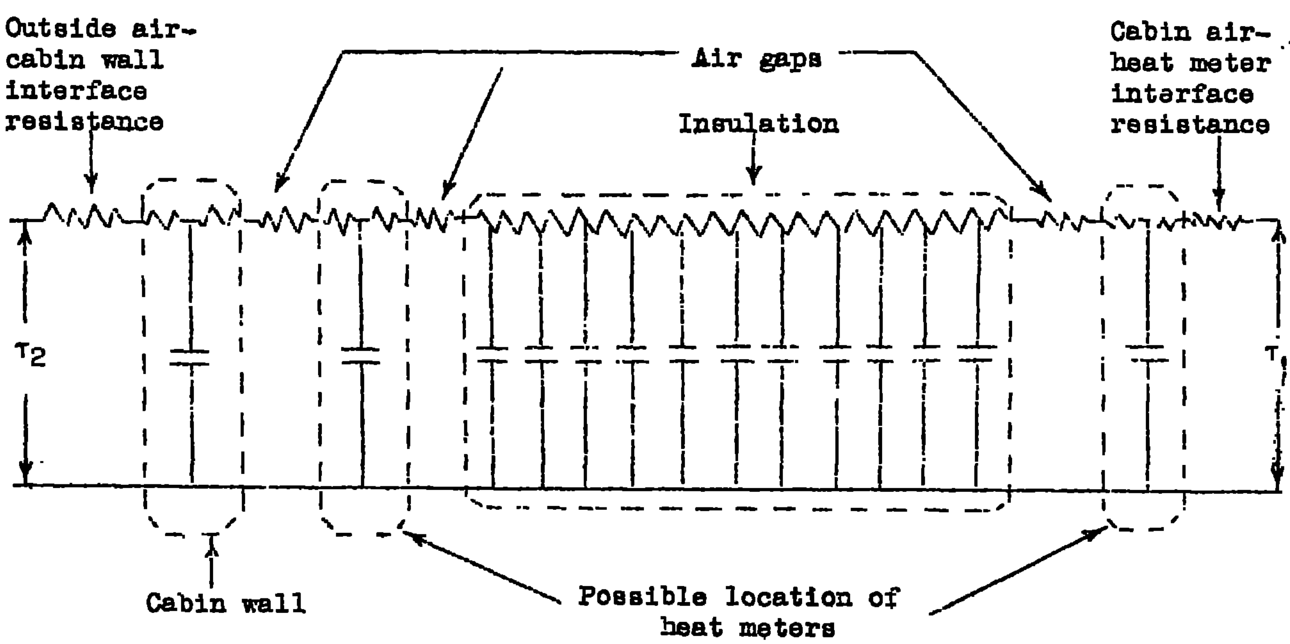
$$\gamma = 5 \text{ lb/ft}^3$$

$$\text{Thickness} = 0.5 \text{ inch}$$

$$c_p = 0.40 \text{ Btu/lb } ^\circ F$$

$$a = \frac{k}{c_p \gamma} = \frac{0.025}{0.40 \times 5} = 0.0125 \text{ ft}^2/\text{hr}$$

Thermal Circuit



The Schmidt solution (fig. 10) was carried out as in the previous case. The insulation was divided into eight small slabs; $\Delta x = 0.0052$ foot and the time increment was

$$\Delta\theta = \frac{(0.0052)^2}{2(0.0125)} = 0.001085 \text{ hour or } 0.065 \text{ minute.}$$

Tables 11 and 12 and figure 11 exhibit the time-heat flow history of the heat meter mounted on the cabin wall as denoted above. The times required for the heat flow into the insulating material as measured by the heat meter from the cabin and the heat flow out of the cabin wall into the outside air to be within 5 percent of the steady state heat flow value after the plane has leveled off, are tabulated below:

Heat flow into insulation (min)	Heat flow out of wall (min)
3.9	4.2

CONCLUSIONS

1. The analytical and graphical solutions of the particular transient heat transfer problems presented in this report may be used as a guide in analyzing transient heat transfer problems in general.

2. If a heat meter is used in a thermal system where any of the quantities such as τ_a, τ_c, or f_c vary in some <u>unknown</u> manner with time so that it is impossible to obtain analytical or graphical solutions, the heat transfer rates as measured by the heat meter cannot be readily interpreted.

3. In the case of a heat meter suddenly applied to a hot surface of constant and uniform temperature, a heat meter reading cannot be made until steady state obtains (when the temperature is constant with time). The time required for steady state to occur can be predicted analytically or graphically.

4. In the case of a heat meter mounted upon the cabin wall of an airplane and the temperature of the outside air decreasing linearly with time, analytical and graphical solutions reveal that large errors in determining rates of heat transfer can be made by taking readings during the transient and considering them as steady state. For the specific case

presented in this report, the difference between the steady state and transient heat transfer is directly proportional to the rate of change (α) of the ambient air temperature and is given by:

$$\left(\frac{q}{A}\right)_{\text{steady state}} - \left(\frac{q}{A}\right)_{\text{transient}} = -0.038\ \alpha$$

5. For an airplane cabin with an appreciable amount of insulation the thermal circuit is difficult to solve by analytical techniques presented because the system contains relatively large distributed resistances and capacitances which do not lend themselves to the analytical method of lumping circuit constants. For those cases the Schmidt method can be used. A knowledge of the thermal capacitances of the insulation may be obtained experimentally (in flight) by using heat meters in the thermal circuit; one could be placed on the side of the insulation contiguous to the cabin air, and the other could be placed between the insulation and the cabin wall. The amount of heat stored in the insulation may be calculated from the difference in the two heat meter readings.

University of California,
Berkeley, Calif., May 1944.

APPENDIX A

THE SCHMIDT (GRAPHICAL) METHOD OF SOLUTION OF THE CONDUCTION EQUATION FOR TRANSIENT UNIDIRECTIONAL HEAT FLOW

Heat conduction problems that are very difficult to solve analytically may be solved graphically by the Schmidt method. The differential equation for unidirectional heat flow in a solid is

$$\frac{\partial t}{\partial \theta} = a \frac{\partial^2 t}{\partial x^2}$$

where t is the temperature at any point, θ the time, x the distance from one face, and a the invariable thermal diffusivity of the slab. The graphical integration of the above partial differential equation is accomplished by replacing the differential quantities by small finite differences. (See references 5, 6, and 7.) Let $\Delta\theta$ represent a small but finite increment of time θ, and let Δx represent a small but finite increment of distance x. The distance increments will be denoted by the subscripts $(n-1)$, n, $(n+1)$, and so forth, and the time increments will be denoted by the subscripts $(m-1)$, m, $(m+1)$, and so forth. Thus $t_{n,m}$ will refer to the temperature of the slab at the distance $n\Delta x$ from the surface at the end of $m\Delta\theta$ time units. Thus the differential quantities are represented in finite difference form as follows:

$$\frac{\Delta t}{\Delta \theta} = \frac{t_{n,(m+1)} - t_{n,m}}{\Delta \theta}$$

The temperature gradient after $m\Delta\theta$ time units between the n^{th} and $(n+1)^{th}$ increment is,

$$\left(\frac{\Delta t}{\Delta x}\right)_+ = \frac{t_{(n+1),m} - t_{n,m}}{\Delta x}$$

The temperature gradient after $m\Delta\theta$ time units between the $(n-1)^{th}$ and n^{th} increment is,

$$\left(\frac{\Delta t}{\Delta x}\right)_- = \frac{t_{n,m} - t_{(n-1),m}}{x}$$

The rate of change of the temperature gradient at the center of the n^{th} layer is,

$$\frac{\Delta^2 t}{\Delta x^2} = \frac{t_{(n+1),m} + t_{(n-1),m} - 2t_{n,m}}{\Delta x^2}$$

When the finite difference quantities are substituted into the conduction equation, the following equation results:

$$\frac{t_{n,(m+1)} - t_{n,m}}{\Delta\theta} = a\left(\frac{t_{(n+1),m} + t_{(m-1),m} - 2t_{n,m}}{\Delta x^2}\right)$$

or

$$t_{n,(m+1)} - t_{n,m} = \frac{2a\ \Delta\ \theta}{\Delta x^2}\left[\frac{t_{(n+1),m} + t_{(n-1),m}}{2} - t_{n,m}\right]$$

If Δx and $\Delta\theta$ are so chosen that

$$\frac{2a\ \Delta\ \theta}{\Delta x^2} = 1,$$

$$t_{n,(m+1)} = \frac{t_{(n+1),m} + t_{(n-1),m}}{2}$$

The above equation states that the temperature of any distance increment at any time is equal to the arithmetic mean of the two distance increments on either side at the previous $\Delta\theta$.

The distance increment is chosen on the basis of the initial temperature distribution. For sudden temperature changes the distance increments should be chosen small enough so that the broken lines connecting the points which represent the temperatures of the distance increments will not differ too greatly from the continuous temperature distribution which would result if distance increments become infinitely small. The corresponding time increment that satisfies the equation, $\frac{2a\ \Delta\ \theta}{\Delta x^2} = 1$, is used.

If the thermal system contains an air-slab interface resistance, a heat rate balance across the boundary must be made so that the temperature at the surface of the slab can be determined.

$$\left(\frac{q}{A}\right)_{x=0} = -k\left(\frac{\partial t}{\partial x}\right)_{x=0} = f(t_s - \tau_a)$$

or

$$-\left(\frac{\partial t}{\partial x}\right)_{x=0} = \frac{(t_s - \tau_a)}{\frac{k}{f}}$$

Thus the slope of the temperature distribution at the boundary is equal to the difference in temperature between the slab surface and the air temperature divided by the ratio k/f. The ratio k/f may vary depending upon the variation of f.

The sudden cooling of an infinite slab with an air-slab interface resistance will be used to illustrate the Schmidt construction. The temperature of the air τ_a, the thermal conductivity of the slab k, the thermal diffusivity of the slab a, and the unit thermal conductance due to convection and radiation f are constant values. The initial temperature of the slab is t_1. In figure 12 the slab is divided into distance increments Δx. If the directrix R, located at ordinate τ_a and at a distance of k/f from the surface of the slab, and the point O (initial temperature t_1 at the surface at $\theta = 0$) are connected by a line RO, the temperature gradient at the surface of the slab is equal to the slope of the line RO. The intersection of line RO with a plane at a distance $\frac{\Delta x}{2}$ from the surface locates a point which may be considered as being on the temperature curve. The following step consists of connecting points a and 2 from which 1' is obtained. Next, R1' is drawn, and 2' is located by connecting points 1 and 3. Thus the temperature distribution after one $\Delta\theta$, 0', 1', 2', 3', 4', and so forth, has been obtained. This procedure is continued for each new $\Delta\theta$.

APPENDIX B

THE SOLUTION OF THE TRANSIENT BEHAVIOR OF A HEAT METER MOUNTED UPON THE CABIN OF AN AIRPLANE IN FLIGHT

The cabin air is maintained at a constant temperature τ_1, and the heat meter, cabin wall, and outside air are initially at this temperature. It is postulated that the temperature of the outside air drops linearly with time. By treating the wall and meter as lumped capacitors and lumping the thermal resistance as described on page 11, it is possible to calculate the temperatures of the heat meter and wall as functions of time. It is also possible to calculate the heat flow from the cabin into the meter, from the meter to the wall, and from the wall to the outside air. For this case the heat flow indicated by the meter is approximately equal to the arithmetical average of the heat flow as calculated into and out of the meter, or $q_m = \dfrac{q_1 + q_2}{2}$.

The solution using lumped parameters is as follows:

A heat balance on the meter yields:

$$q_1 = \frac{\tau_c - t_m}{R_1} = C_m \frac{dt_m}{d\theta} + \frac{t_m - t_w}{R_6} \tag{B1}$$

While a heat balance on the wall yields:

$$q_2 = \frac{t_m - t_w}{R_6} = C_w \frac{dt_w}{d\theta} + \frac{t_w - \tau_2}{R_7} \tag{B2}$$

where

$$\frac{t_w - \tau_2}{R_7} = q_3$$

Adding equations (B1) and (B2), gives:

$$\frac{\tau_c - t_m}{R_1} = C_m \frac{dt_m}{d\theta} + C_w \frac{dt_w}{d\theta} + \frac{t_w - \tau_2}{R_7} \tag{B3}$$

From equation (B1) is obtained:

$$t_w = \left(\frac{R_1 + R_6}{R_1}\right) t_m + R_6\ C_m \frac{dt_m}{d\,\theta} - \frac{R_6}{R_1}\tau_c \qquad \text{(B4)}$$

Differentiating (B4) with respect to θ, gives:

$$\frac{dt_w}{d\,\theta} = R_6\ C_m \frac{d^2t_m}{d\theta^2} + \left(\frac{R_1 + R_6}{R_1}\right)\frac{dt_m}{d\,\theta} \qquad \text{(B5)}$$

Substituting (B4) and (B5) into (B3) gives upon rearrangement:

$$C_m\ C_w \frac{d^2t_m}{d\theta^2} + \left[C_m\left(\frac{R_6 + R_7}{R_6\ R_7}\right) + C_w\left(\frac{R_1 + R_6}{R_1\ R_6}\right)\right]\frac{dt_m}{d\theta}$$

$$+ \left(\frac{R_1 + R_6 + R_7}{R_1\ R_6\ R_7}\right) t_m = \frac{R_6 + R_7}{R_1\ R_6\ R_7} + \frac{\tau_a}{R_6\ R_7}$$

$$= \left(\frac{R_1 + R_6 + R_7}{R_1\ R_6\ R_7}\right)\tau_c - \frac{\alpha\,\theta}{R_6\ R_7} \qquad \text{(B6)}$$

This is the differential equation for the heat meter temperature as a function of time. To simplify this equation, let

$$a = C_m\ C_w \qquad \text{(B7)}$$

$$b = \frac{C_m\ (R_6 + R_7)}{R_6\ R_7} + \frac{C_w\ (R_1 + R_6)}{R_1\ R_6} \qquad \text{(B8)}$$

$$c = \left(\frac{R_1 + R_6 + R_7}{R_1\ R_6\ R_7}\right) \qquad \text{(B9)}$$

Then (B6) is rewritten as follows:

$$a\frac{d^2t_m}{d\theta^2} + b\frac{dt_m}{d\,\theta} + ct_m = \left(\frac{R_1 + R_6 + R_7}{R_1\ R_6\ R_7}\right)\tau_c - \frac{\alpha\,\theta}{R_6\ R_7} \qquad \text{(B10)}$$

And let

$$n_3 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and

$$n_4 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad \text{(B11)}$$

The solution of the reduced equation is

$$t_m = A_1 e^{n_3 \theta} + B_1 e^{n_4 \theta} \qquad \text{(B12)}$$

where A_1 and B_1 are integration constants dependent upon the initial conditions.

The particular solution is

$$t_m = \tau_c - \frac{\alpha R_1 \theta}{R_1 + R_6 + R_7} + \frac{b \alpha R_1(R_1 R_6 R_7)}{(R_1 + R_6 + R_7)^2} \qquad \text{(B13)}$$

and the complete solution is

$$t_m = A_1 e^{n_3 \theta} + B_1 e^{n_4 \theta} + \tau_c - \frac{\alpha R_1 \theta}{R_1 + R_6 + R_7}$$

$$+ \frac{b \alpha R_1(R_1 R_6 R_7)}{(R_1 + R_6 + R_7)^2} \qquad \text{(B14)}$$

The initial conditions for determining A_1 and B_1 are that when $\theta = 0$; $t_m = \tau_1$; $\frac{dt_m}{d\theta} = 0$. From equation (B11)

$$A_1 + B_1 = \frac{b \alpha R_1(R_1 R_6 R_7)}{(R_1 + R_6 + R_7)^2} \qquad \text{(B15)}$$

And from the derivative of equation (B14):

$$n_3\ A_1 + n_4\ B_1 = \frac{\alpha\ R_1}{R_1 + R_6 + R_7} \qquad \text{(B16)}$$

Solving equations (B15) and (B16) for A_1 and B_1 gives:

$$A_1 = \frac{\alpha\ R_1}{(n_3 - n_4)(R_1 + R_6 + R_7)} \left[1 + \frac{b\ n_4(R_1\ R_6\ R_7)}{(R_1 + R_6 + R_7)^2}\right]$$

and

$$B_1 = \frac{\alpha\ R_1}{(n_4 - n_3)(R_1 + R_6 + R_7)} \left[1 + \frac{b\ n_3\ R_1\ R_6\ R_7}{(R_1 + R_6 + R_7)^2}\right]$$

And on substitution into equation (B11) the equation for the temperature variation with time of the heat meter is obtained:

$$t_m = \tau_c - \frac{\alpha\ R_1 \theta}{R_1 + R_6 + R_7} + \frac{b\ \alpha\ R_1(R_1\ R_6\ R_7)}{(R_1 + R_6 + R_7)^2}$$

$$+ \frac{\alpha\ R_1\ e^{n_3\ \theta}}{(n_3 - n_4)(R_1 + R_6 + R_7)} \left[1 + \frac{b\ n_4\ R_1\ R_6\ R_7}{R_1 + R_6 + R_7}\right]$$

$$+ \frac{\alpha\ R_1\ e^{n_4\ \theta}}{(n_4 - n_3)(R_1 + R_6 + R_7)} \left[1 + \frac{b\ n_3\ R_1\ R_6\ R_7}{R_1 + R_6 + R_7}\right] \qquad \text{(B17)}$$

To obtain the equation for the temperature variation of the wall of the airplane a similar method is employed. The differential equation is nearly the same as before and is

$$a\ \frac{d^2 t_w}{d\theta^2} + b\ \frac{dt_w}{d\,\theta} + ct_w = \left(\frac{R_1 + R_6 + R_7}{R_1\ R_6\ R_7}\right) \tau_c - \frac{(R_1 + R_6)\alpha\,\theta}{R_1\ R_6\ R_7} - \frac{C_m \alpha}{R_7} \qquad \text{(B18)}$$

where a, b, and c are the same as in equation (B6).

The solution of the equation is then

$$t_w = A_2\, e^{n_3\,\theta} + B_2\, e^{n_4\,\theta} + \tau_c - \frac{(R_1 + R_6)\,\alpha\,\theta}{R_1 + R_6 + R_7}$$

$$- \frac{C_m\,\alpha\,R_1\,R_6}{R_1 + R_6 + R_7} + \frac{b\,\alpha(R_1 + R_6)R_1\,R_6\,R_7}{(R_1 + R_6 + R_7)^2} \qquad \text{(B19)}$$

The initial conditions are, when $\theta = 0$; $t_w = \tau_c$; $\frac{dt_w}{d\theta} = 0$ so that from equation (B16)

$$A_2 + B_2 = \frac{C_m\,\alpha\,R_1\,R_6}{R_1 + R_6 + R_7} - \frac{b\,\alpha(R_1 + R_6)R_1\,R_6\,R_7}{(R_1 + R_6 + R_7)^2} \qquad \text{(B20)}$$

and

$$n_3\,A_2 + n_4\,B_2 = \left(\frac{R_1 + R_6}{R_1 + R_6 + R_7}\right)\alpha \qquad \text{(B21)}$$

This gives the equation for t_w as:

$$t_w = \tau_c - \frac{(R_1 + R_6)\alpha\,\theta}{R_1 + R_6 + R_7} - \frac{C_m\,\alpha\,R_1\,R_6}{R_1 + R_6 + R_7} + \frac{b\,\alpha(R_1 + R_6)(R_1\,R_6\,R_7)}{(R_1 + R_6 + R_7)^2}$$

$$+ \frac{\alpha\,(R_1 + R_3)\,e^{n_3\,\theta}}{(n_3 - n_4)(R_1 + R_6 + R_7)}\left[1 + \frac{b\,n_4\,R_1\,R_6\,R_7}{R_1 + R_6 + R_7} - \frac{n_4\,C_m\,R_1\,R_6}{R_1 + R_6}\right]$$

$$+ \frac{\alpha\,(R_1 + R_3)\,e^{n_4\,\theta}}{(n_3 - n_1)(R_1 + R_6 + R_7)}\left[1 + \frac{b\,n_3\,R_1\,R_6\,R_7}{R_1 + R_6 + R_7} - \frac{n_3\,C_m\,R_1\,R_6}{R_1 + R_6}\right] \qquad \text{(B22)}$$

Thus far the equations for t_m, the heat meter temperature, and t_w the cabin wall temperature have been obtained. To calculate the heat flow as read by the heat meter it is necessary to obtain q_1 and q_2, or the heat flow into and out of the meter. From equations (B1) and (B2):

$$q_1 = \frac{T_c - t_m}{R_1} \tag{B1}$$

and

$$q_2 = \frac{t_m - t_w}{R_6} \tag{B2}$$

From equations (B1) and (B17)

$$q_1 = \frac{\alpha\,\theta}{R_1 + R_6 + R_7} - \frac{b\,\alpha\,R_1\,R_6\,R_7}{(R_1 + R_6 + R_7)^2} - \frac{\alpha\,e^{n_3\theta}}{(n_3 - n_4)(R_1 + R_6 + R_7)}$$

$$\left[1 + \frac{b\,n_4\,R_1\,R_6\,R_7}{R_1 + R_6 + R_7}\right] - \frac{\alpha\,e^{n_4\theta}}{(n_4 - n_3)(R_1 + R_6 + R_7)}\left[1 + \frac{b\,n_3 R_1\,R_6\,R_7}{R_1 + R_6 + R_7}\right] \tag{B23}$$

and from equations (B2) and (B14) and (B19):

$$q_2 = -\frac{\alpha R_1 \theta}{R_6(R_1 + R_6 + R_7)} + \frac{(R_1 + R_6)\alpha\theta}{R_6(R_1 + R_6 + R_7)} + \frac{b\alpha R_1(R_1 R_6 R_7)}{R_6(R_1 + R_6 + R_7)^2}$$

$$-\frac{b\alpha(R_1 + R_6)R_1 R_6 R_7}{R_6(R_1 + R_6 + R_7)^2} + \frac{C_m \alpha R_1}{R_1 + R_6 + R_7} + \frac{\alpha R_1 e^{n_3\theta}}{R_6(n_3 - n_4)(R_1 + R_6 + R_7)}$$

$$\left[1 + \frac{bn_4 R_1 R_6 R_7}{R_1 + R_6 + R_7}\right] - \frac{\alpha(R_1 + R_6)e^{n_3\theta}}{R_6(n_3 - n_4)(R_1 + R_6 + R_7)}\left[1 + \frac{R_1 R_6 R_7}{R_1 + R_6 + R_7}\right]$$

$$+\frac{\alpha C_m n_4 e^{n_3\theta} R_1}{(n_3 - n_4)(R_1 + R_6 + R_7)} + \frac{\alpha R_1 e^{n_4\theta}}{R_6(n_4 - n_3)(R_1 + R_6 + R_7)}$$

$$\left[1 + \frac{bn_3 R_1 R_6 R_7}{R_1 + R_6 + R_7}\right] - \frac{\alpha(R_1 + R_6)e^{n_4\theta}}{R_6(n_4 - n_3)(R_1 + R_6 + R_7)}$$

$$\left[1 + \frac{bn_3 R_1 R_6 R_7}{R_1 + R_6 + R_7} - \frac{n_3 C_m R_1 R_6}{R_1 + R_6}\right] \tag{B24}$$

which reduces to:

$$q_2 = \frac{\alpha\theta}{R_1 + R_6 + R_7} - \frac{b\alpha R_1 R_6 R_7}{(R_1 + R_6 + R_7)^2} + \frac{C_m \alpha R_1}{R_1 + R_6 + R_7}$$

$$-\frac{\alpha e^{n_3\theta}}{(n_3 - n_4)(R_1 + R_6 + R_7)}\left[1 + \frac{bn_4 R_1 R_6 R_7}{R_1 + R_6 + R_7} - C_m R_1 n_4\right]$$

$$-\frac{\alpha e^{n_4\theta}}{(n_4 - n_3)(R_1 + R_6 + R_7)}\left[1 + \frac{bn_3 R_1 R_6 R_7}{R_1 + R_6 + R_7} - C_m R_1 n_3\right] \tag{B25}$$

and since

$$q_m = \frac{q_1 - q_2}{2}$$

$$q_m = \frac{\alpha\,\theta}{R_1 + R_6 + R_7} - \frac{b\,\alpha\,R_1\,R_6\,R_7}{(R_1 + R_6 + R_7)^2} + \frac{C_m\,\alpha\,R_1}{2(R_1 + R_6 + R_7)}$$

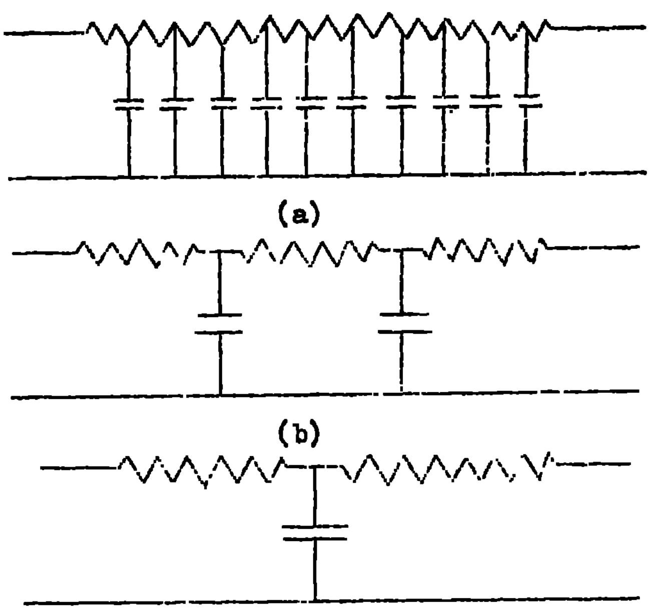
$$- \frac{\alpha\,e^{n_3\,\theta}}{(n_3 - n_4)(R_1 + R_6 + R_7)}\left[1 + \frac{bn_4\,R_1\,R_6\,R_7}{R_1 + R_6 + R_7} - \frac{C_m\,R_1\,n_4}{2}\right]$$

$$- \frac{\alpha\,e^{n_4\,\theta}}{(n_4 - n_3)(R_1 + R_6 + R_7)}\left[1 + \frac{bn_3\,R_1\,R_6\,R_7}{R_1 + R_6 + R_7} - \frac{C_m\,R_1\,n_3}{2}\right]$$

(B26)

APPENDIX C

A distributed resistance and capacitance in a thermal system can be idealized so that the distributed resistance may be combined into one or more resistors, and the distributed capacitance may be combined into one or more capacitors. This technique of combining the distributed resistance and capacitance will yield a system containing lumped circuit parameters.



Distributed resistance and capacitance

(a)

Lumped resistances and capacitances

(b)

(c)

For instance, diagrams b and c are examples of elements which have been obtained by lumping the distributed resistance and capacitance of element a. The degree to which diagram a would be simplified (b or c) would depend upon the elements of the complete system and upon the accuracy desired.

REFERENCES

1. Martinelli, R. C., Morrin, E. H., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. V - Theory and Use of Heat Meters for the Measurement of Rates of Heat Transfer Which Are Independent of Time. NACA ARR, Dec. 1942.

2. Boelter, L. M. K., Poppendiek, H. F., and Gier, J. T.: An Investigation of Aircraft Heaters. XVII - Experimental Inquiry into Steady State Unidirectional Heat-Meter Corrections. NACA ARR No. 4H09, 1944.

3. Brombacher, W. G.: Altitude-Pressure Tables Based on the United States Standard Atmosphere. NACA Rep. No. 538 (Reprint), 1938, Table III, p. 13.

4. Martinelli, R. C., Guibert, A. G., Morrin, E. H., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. VIII - A Simplified Method for the Calculation of the Unit Thermal Conductance over Wings. NACA ARR, March 1943, equation (8).

5. Schmidt, Ernst: Einfuhrung in die Technische Thermodynamik, J. Springer (Berlin), 1936, pp. 26?-268.

6. Sherwood, T. K., and Reed, C. E.: Applied Mathematics in Chemical Engineering. McGraw-Hill Book Co., Inc. (New York), 1939.

7. Nessi, A., and Nissolle, L.: Physique Industrielle, Methodes Graphiques Pour l'Etude des Installations de Chauffage et de Refrigeration en Regime Discontinu. Dunod (Paris), 1929.

8. Reddick, H. W., and Miller, F. H.: Advanced Mathematics for Engineers. John Wiley & Sons, Inc. (New York), 1938.

9. Ford, L. R.: Differential Equations. McGraw-Hill Book Co., Inc., 1933.

TABLE I

PROPERTIES OF ELEMENTS IN THE THERMAL SYSTEM FOR CASE I

	Thermal Conductivity, k, Btu/hr ft^2($^\circ$F/ft)	Heat Capacity, c_p Btu/lb $^\circ$F	Weight Density, γ lbs/ft^3	Thickness, ft
Air Gap	0.016	$\gamma c_p \cong 0$		0.000755
Heat Meter	0.13	0.35	89.3	0.004

$(f_c + f_r) = 2.0$ Btu/hr ft^2 $^\circ$F at steady state

TABLE II

Results of Analytical Solution

(Eq. 5, Fig. 1)

Time minutes	Heat Meter Temperature $^\circ$F
0	78
0.25	95.5
0.50	105.7
0.75	111.6
1.00	115.1
1.50	118.3
2.00	119.4
∞	119.9

TABLE III

Results of Graphical Schmidt Solution

(Fig. 2)

Time minutes	Heat Meter Temperature $^\circ$F
0	78
.04	80
.10	85.6
.22	94
.33	100.4
.56	108.4
.91	114.2
1.37	117
1.60	117.5
1.94	118
2.17	118.2
∞	119.9

Analytical Time-Heat Flow History of a Heat Meter Placed upon a Hot Surface, See Fig. 3

(Equation 7, Page 11)

Time Min.	$\frac{q_M}{q_{M\infty}}$	$\frac{q_M}{A}$ Btu/hr ft^2
0	0	0
0.01	1.50	124
0.02	2.50	207
0.03	3.16	262
0.05	3.84	318
0.07	4.08	338
0.10	4.08	338
0.15	3.80	315
0.20	3.48	288
0.25	3.12	258
0.30	2.92	242
0.50	2.15	178
0.70	1.68	139
1.00	1.32	109
1.50	1.09	90

TABLE V

Graphical Results by the Schmidt Method of the Heat Flow-Time History of a Heat Meter Suddenly Placed upon a Hot Surface

(see Figs. 2 and 3)

Time Min.	t_1 °F	t_2 °F	t_1-t_2 °F	q_M/A Btu/hr ft^2
0	78.0	78.0	0.0	0
.0072	78.4	78.0	0.4	39.0
.0144	79.5	78.0	1.5	146.0
.0216	80.8	78.4	2.4	234.0
.0288	81.9	79.0	2.9	283.0
.0360	82.8	79.6	3.2	312.0
.0432	83.7	80.2	3.5	342.0
.0720	86.6	82.9	3.7	361.0
.1008	89.0	85.6	3.4	332.0
.1296	91.1	87.9	3.2	312.0
.1584	93.2	90.1	3.1	302.0
.1876	95.0	92.1	2.9	283.0
.2164	96.7	94.0	2.7	264.0
.2452	98.4	95.8	2.6	254.0
.2740	99.9	97.4	2.5	244.0
.3028	101.1	98.8	2.3	224.0
.3316	102.5	100.3	2.2	214.0
.447	106.6	104.8	1.8	176.0
.562	110.0	108.4	1.6	156.0
.677	112.3	111.0	1.3	127.0
.792	114.0	112.9	1.1	107.0
.907	115.2	114.1	1.1	107.0
1.022	116.1	115.1	1.0	97.6
1.366	117.8	116.9	0.9	87.8
∞	120.75	119.9	0.85	82.8

TABLE VI

Properties of the Atmosphere

Elev. ft.	Time of Flight minutes	Air Temp. °F	Air Density, γ lb/ft^3	Air Press. lb/in^2 abs.
0	0	60	0.0765	14.7
3000	1.00	48.3	0.0700	13.17
5000	1.67	41.2	0.0659	12.23
10,000	3.33	23.3	0.0565	10.11
12,000	4.00	16.2	0.0530	9.35
15,000	5.00	5.51	0.0481	8.29
20,000	6.66	-12.3	0.0408	6.75
22,000	7.33	-19.5	0.0281	6.20
25,000	8.33	-30.2	0.0343	5.45
30,000	10.00	-48.0	0.0286	4.36
35,000	11.67	-65.8	0.0237	3.46
40,000	13.32	-67.0	0.0234	2.72

TABLE VII

The evaluation of f_c as a function of altitude and time

elevation feet	time min.	T_2 °F	T °R	U_m ft/sec	γ_{air} lb/ft^3	$\left(\frac{U_m \gamma_{air}}{L^{0.25}}\right)^{0.80}$	$T^{0.3}$	f_c Btu/hr ft^2 °F	$\frac{k}{f_c}$
0	0	60	520	146	.0765	4.82	6.52	20.0	0.0041
3000	1	48.5	508.4	146	.0702	4.50	6.50	18.65	0.00443
6000	2	37.5	497.5	146	.0641	4.19	6.45	17.5	0.00471
12000	4	16	476	146	.0532	3.60	6.35	14.6	0.00565
18000	6	-5.5	454.5	146	.0435	3.06	6.30	12.25	0.00673
24000	8	-27.0	433	146	.0360	2.58	6.20	10.2	0.00815
30000	10	-48.0	412	146	.0284	2.18	6.10	8.49	0.00971

$$f_c = 0.64 \; T^{0.3} \left(\frac{U_m \gamma}{L^{0.25}}\right)^{0.80} \qquad \text{equation (8)}$$

f_r on outside is neglected compared to f_c

TABLE VIII

PROPERTIES OF THE SYSTEM FOR CASE II

element	k Btu/hr ft^2 $\frac{°F}{ft}$	thickness ft	γ lb/ft^3	c_p Btu/°F lb
heat meter	0.0824	0.004	(1.43)(62.4) = 89.3	0.35
air gap	0.016	0.000 533	$\gamma c_p \cong 0$	
aluminum cabin wall	116	0.00267	165	0.21

Analytical Results of the Heat Flow-Time History of a Heat Meter Mounted upon the Inside of the Cabin Wall of an Airplane in Flight (Equation 34, Figure 3)

θ hours	θ minutes	$\frac{q_M}{A}$ $\frac{\text{Btu}}{\text{hr ft}^2}$
0	0	0
.005	0.3	5.56
.01	0.6	13.3
.015	0.9	20.6
.02	1.2	26.2
.03	1.8	36.1
.04	2.4	44.0
.05	3.0	51.0
.075	4.5	66.5
.10	6.0	81.0
.125	7.5	95.0
.15	9.0	109.0

θ hours	θ minutes	$\frac{q_M}{A}$ $\frac{\text{Btu}}{\text{hr ft}^2}$
0.155	9.3	111.7
.157	9.42	111.1
.160	9.6	108.3
.162	9.72	106.3
.165	9.9	103.5
.170	10.2	99.5
.175	10.5	96.5
.185	11.1	92.4
.195	11.7	90.1
.205	12.3	88.8
.235	14.1	87.45
.255	15.3	87.09

TABLE X

The Heat Flow-Time History of a Heat Meter

Mounted upon the Inside of the Cabin Wall of an Airplane in Flight

by the Schmidt Method

θ hrs	t_2 °F	t_1 °F	$t_2 - t_1$ °F	$\frac{q_M}{A}$ Btu/hr ft^2
0	60.0	60.0	0	0
.00303	60.0	59.85	0.15	9.2
.00606	59.2	59.0	0.20	12.3
.00909	58,30	58.0	0.30	18.4
.01212	57.0	56.6	0.40	24.6
.01515	55.8	55.25	0.55	33.7
.303	48.3	47.6	0.70	42.9
.0485	38.45	37.6	0.85	52.1
.0606	31.85	30.9	0.95	58.3
.0909	15.2	14.0	1.20	73.5
.1212	-0.8	-2.2	1.40	85.8
.1545	-17.6	-19.25	1.65	101.2
.1575	-19.0	-20.7	1.70	104.0
.161	-20.1	-21.7	1.60	98.0
.1635	-21.0	-22.55	1.55	95.0
.1665	-21.75	-23.25	1.50	91.9
.170	-22.35	-23.80	1.45	88.9
.173	-22.80	-24.2	1.45	88.9
∞	-25.20	-26.55	1.35	83.0

Results by the Schmidt Method of Heat Transfer out of the Composite Cabin Wall

Into the Outside Air as Measured by the Heat Meter

As a Function of Time

θ Min.	$\frac{q_M}{A}$ Btu/hr ft^2
0	0
.065	3.85
.130	5.77
.195	7.70
.455	9.62
.715	14.42
.975	17.30
1.235	21.1
1.495	22.1
1.755	23.55
2.275	29.3
2.795	32.2
3.315	35.1
4.355	39.4
5.395	43.25
6.435	48.1
7.475	53.4
8.515	55.8
9.295	58.2
9.555	52.9
9.815	50.5
10.075	47.6
10.335	45.7
11.895	40.4
∞	35.1

TABLE XII

Results by the Schmidt Method of Heat Transfer into the Composite Cabin Wall

From the Cabin as Measured by the Heat Meter

As a Function of Time

θ Min.	$\frac{q_M}{A}$ Btu/hr ft^2
0	0
.065	0
.130	0
.195	0
.455	0
.715	0
.975	0
1.235	0.48
1.495	0.48
1.755	1.44
2.275	1.92
2.795	2.885
3.315	3.85
4.355	7.69
5.395	11.05
6.435	14.90
7.475	18.25
8.515	23.10
9.295	25.97
9.555	26.45
9.815	27.40
10.075	28.35
10.335	29.30
10.595	30.3
10.855	31.3
11.115	31.7
11.375	32.2
11.635	33.2
11.895	33.2
∞	35.6

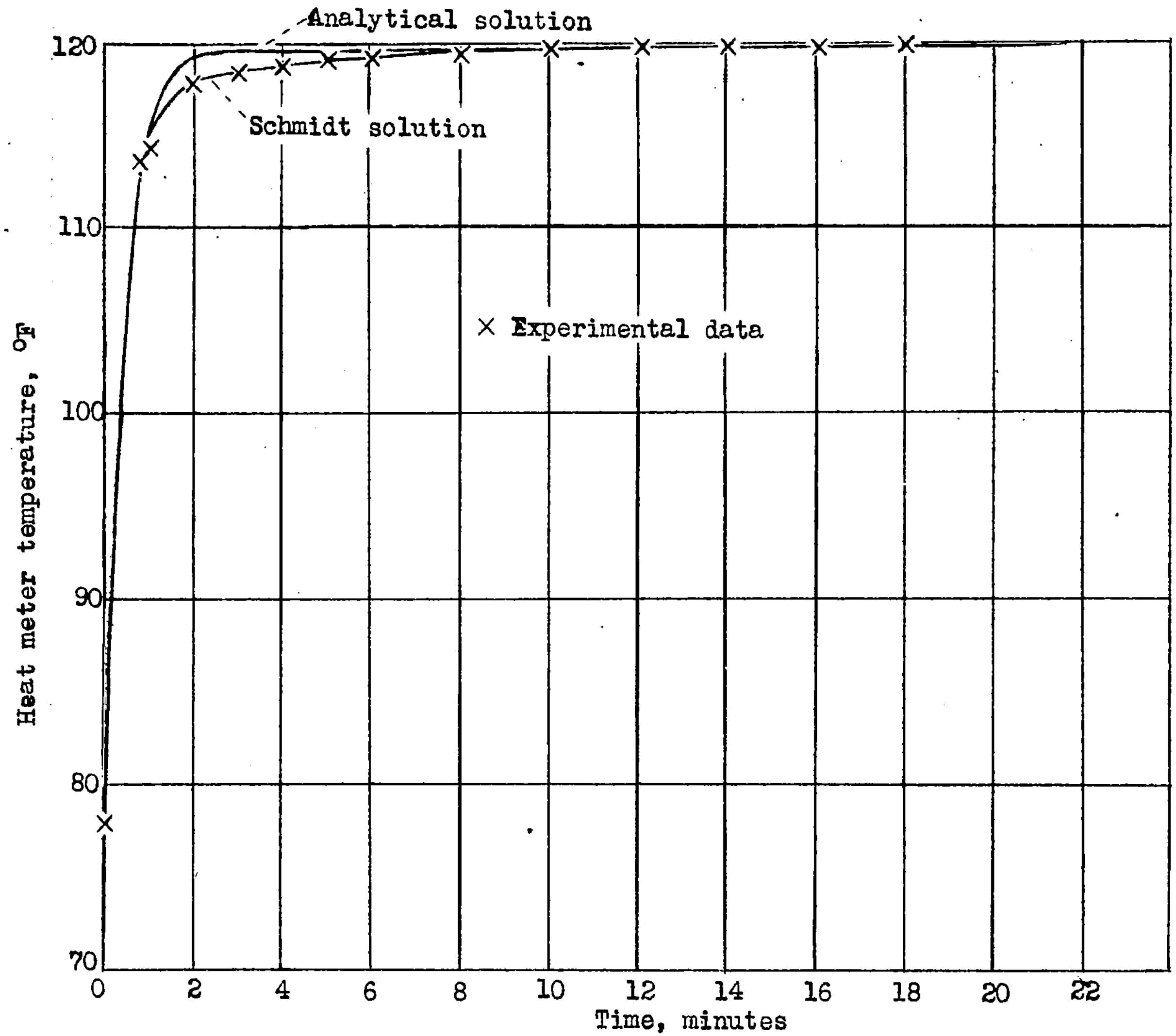


Figure 1.- Time-temperature history of heat meter suddenly placed on a hot surface.

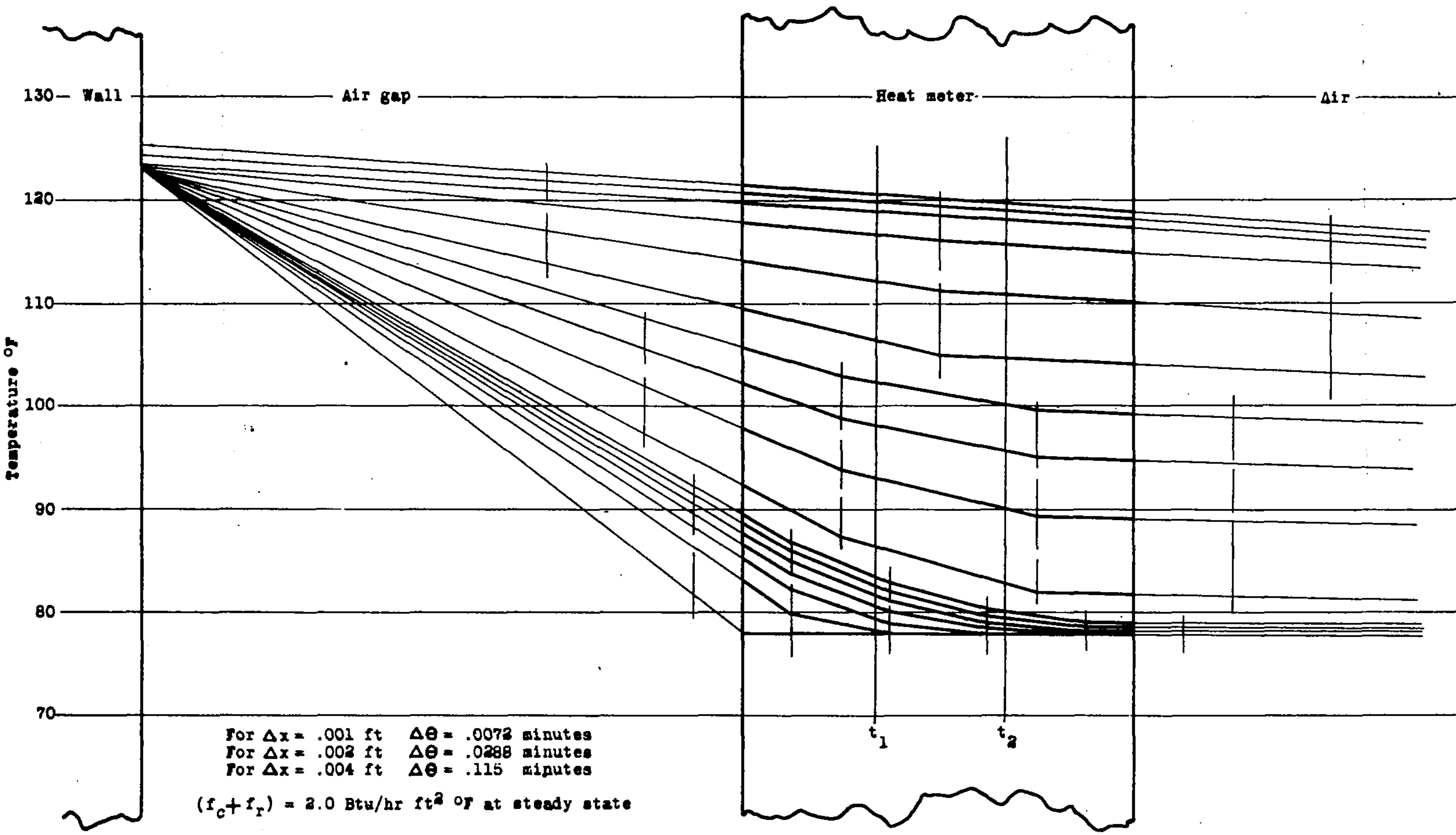


Figure 2.- The Schmidt solution of a heat meter suddenly placed upon a hot surface.

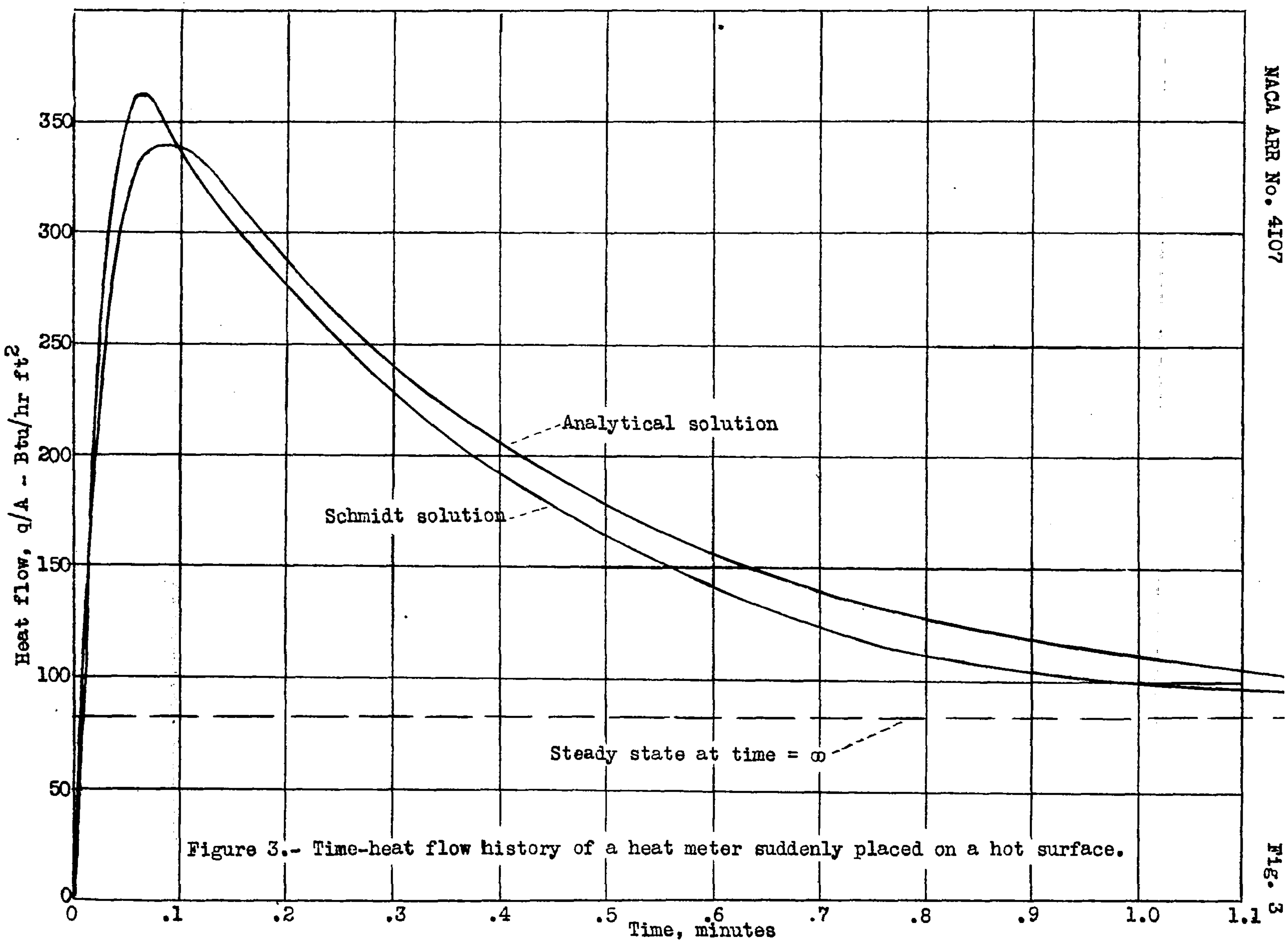


Figure 3.- Time-heat flow history of a heat meter suddenly placed on a hot surface.

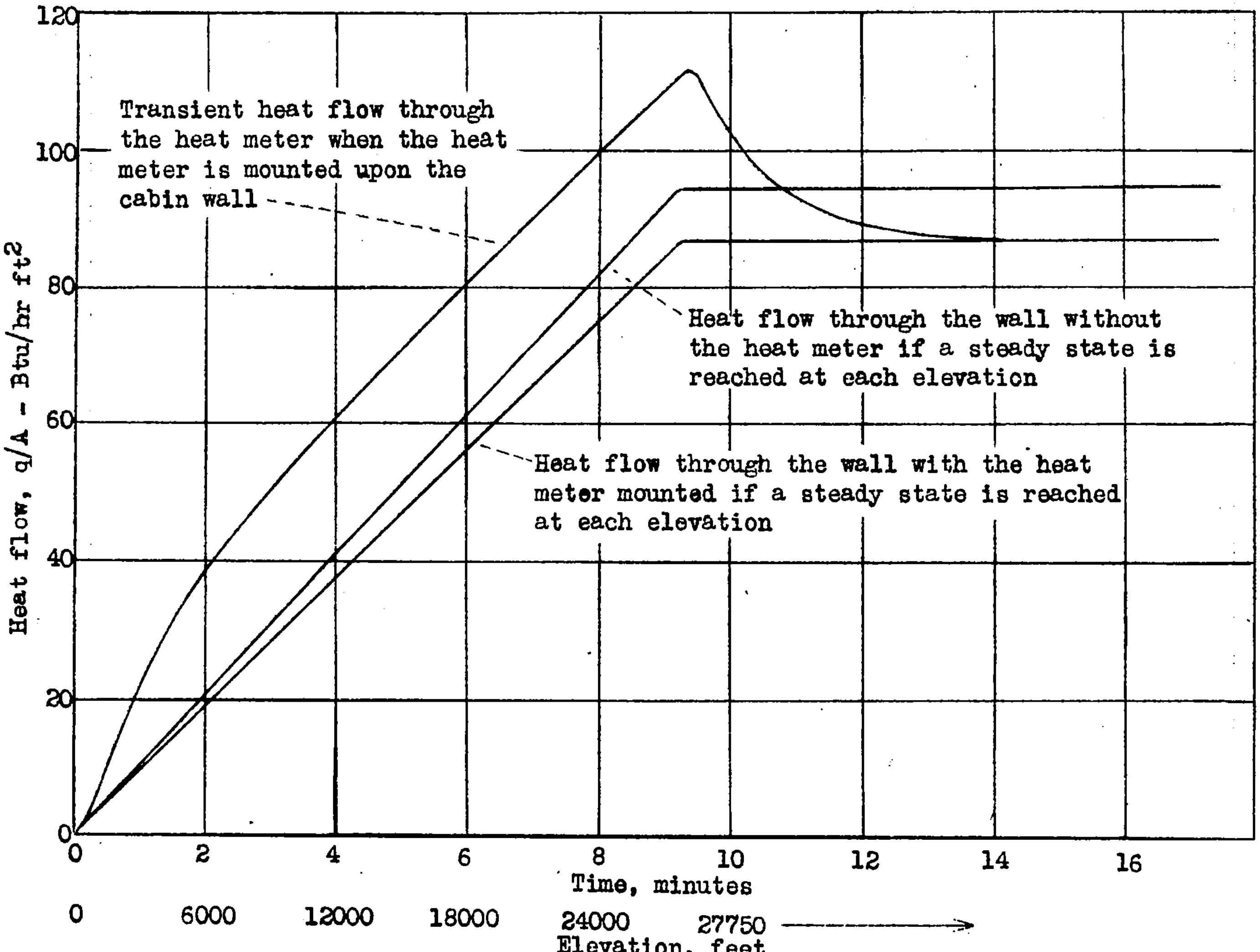


Figure 4.- The analytical solution of the time-heat flow history of a heat meter mounted on the inside of the cabin wall of an airplane in flight.

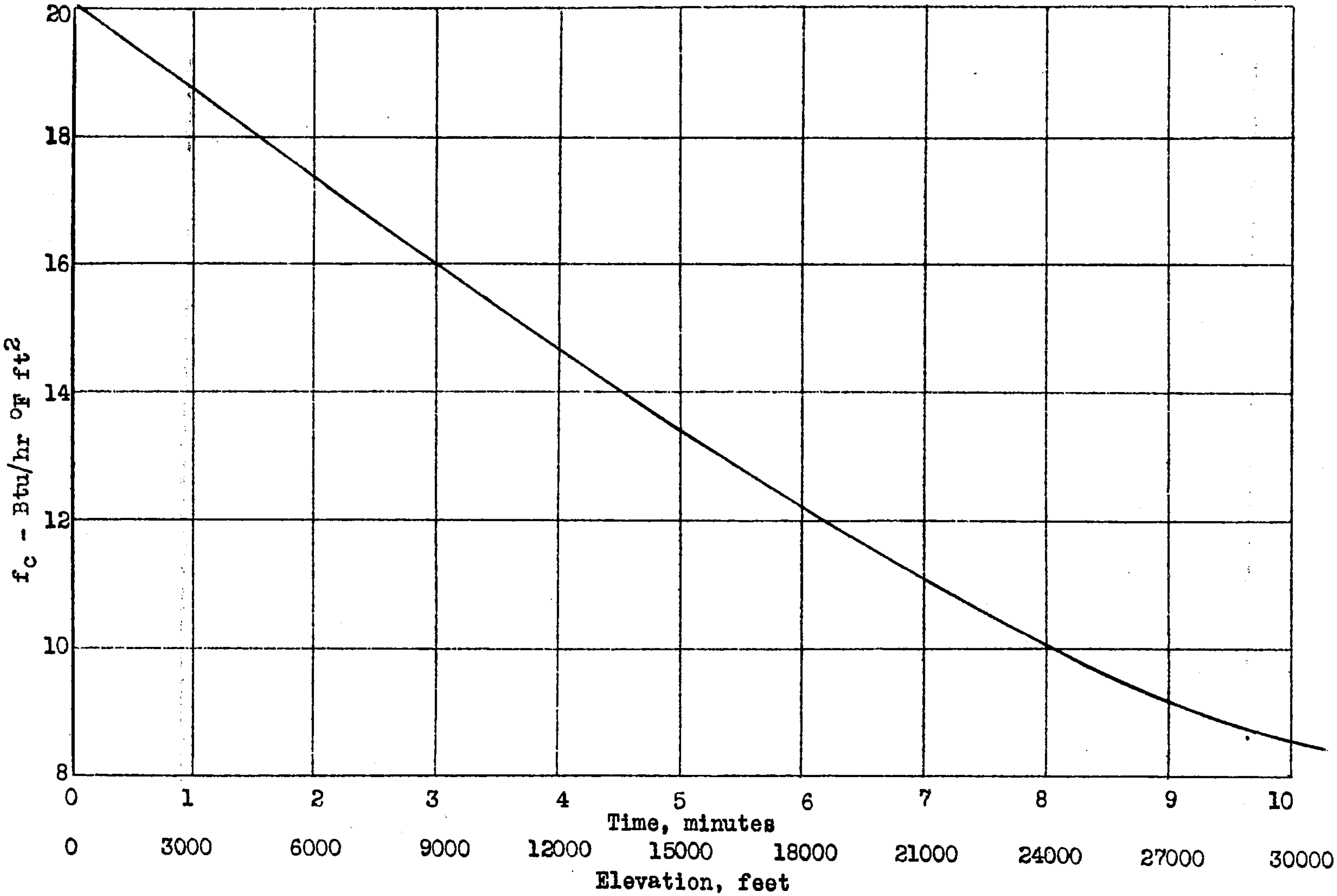


Figure 5.- The outside thermal conductance due to forced convection on the cabin wall as a function of time.

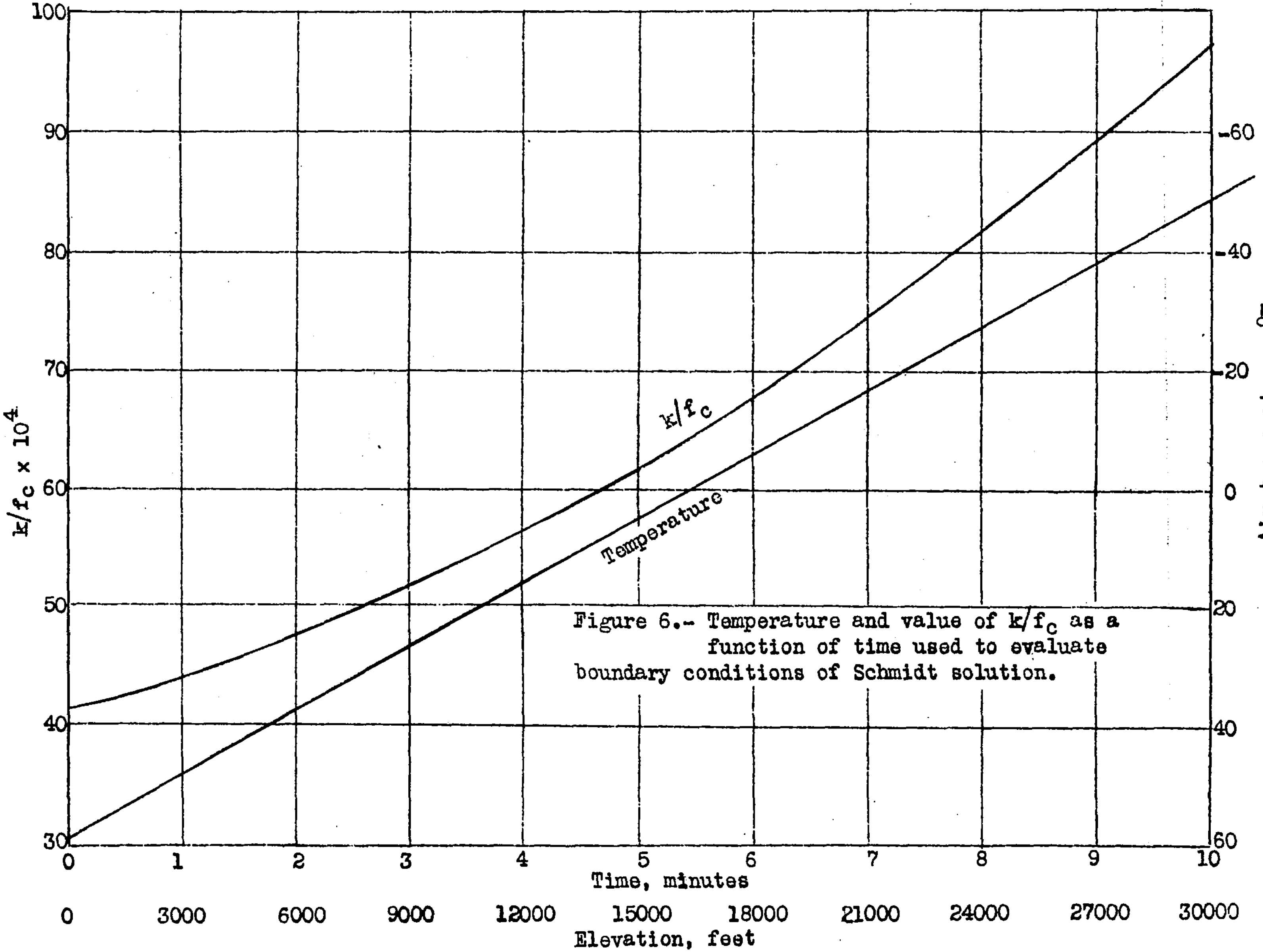


Figure 6.- Temperature and value of k/f_c as a function of time used to evaluate boundary conditions of Schmidt solution.

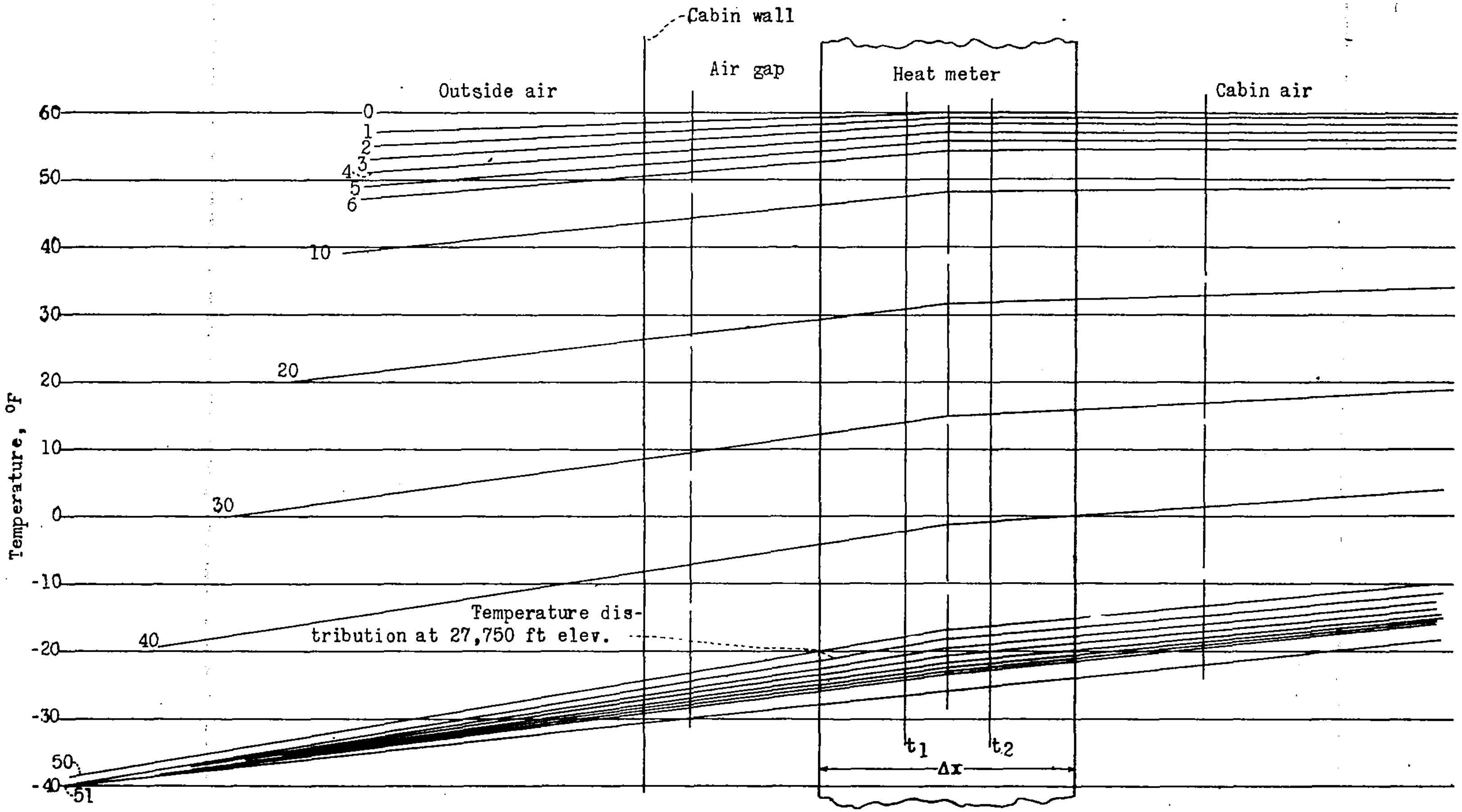


Figure 7.- The Schmidt solution for a heat meter mounted on the inside of an uninsulated cabin wall of an airplane in flight.

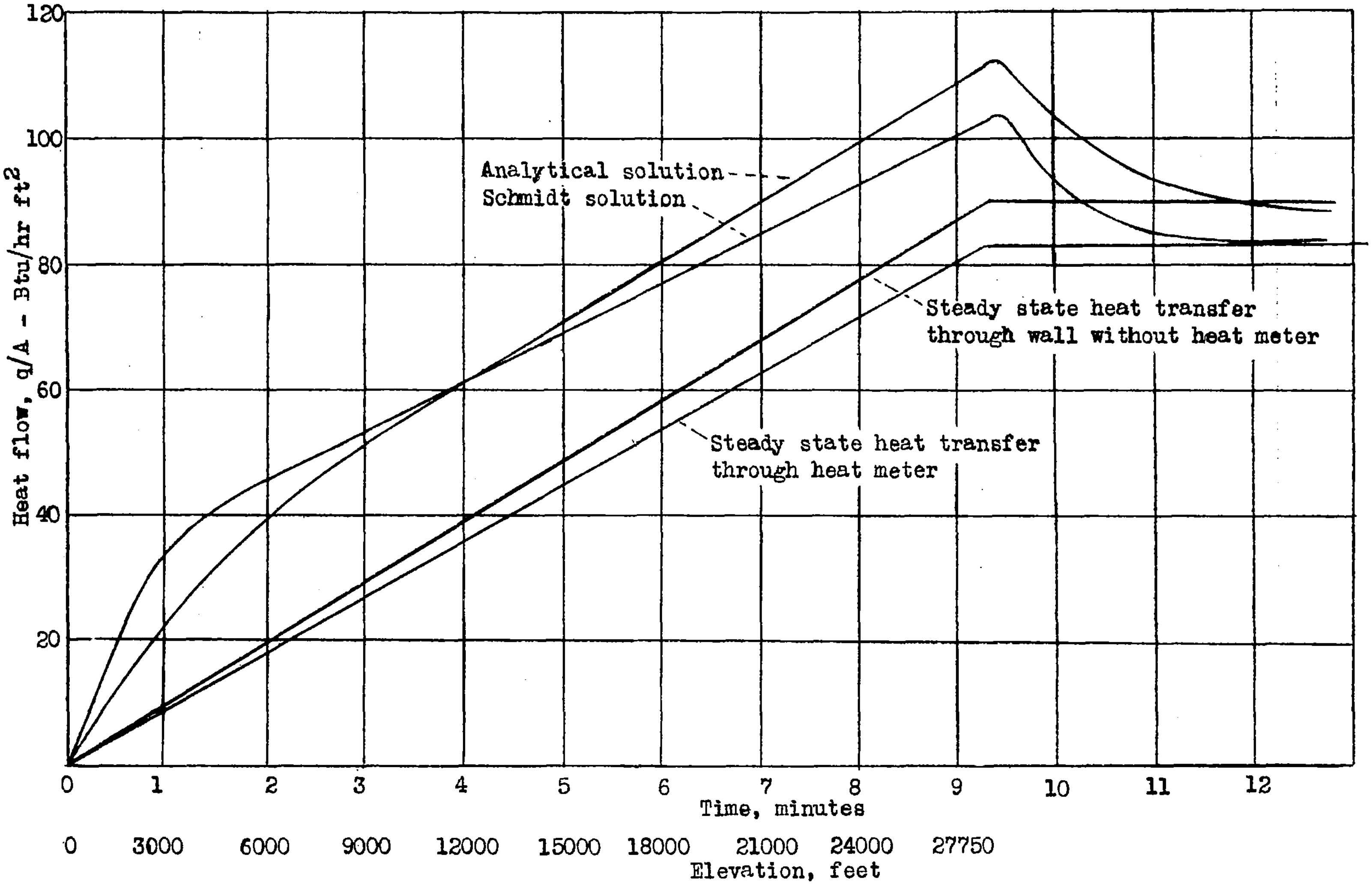


Figure 8.- The time-heat flow history of a heat meter mounted on the inside of the cabin wall of an airplane in flight.

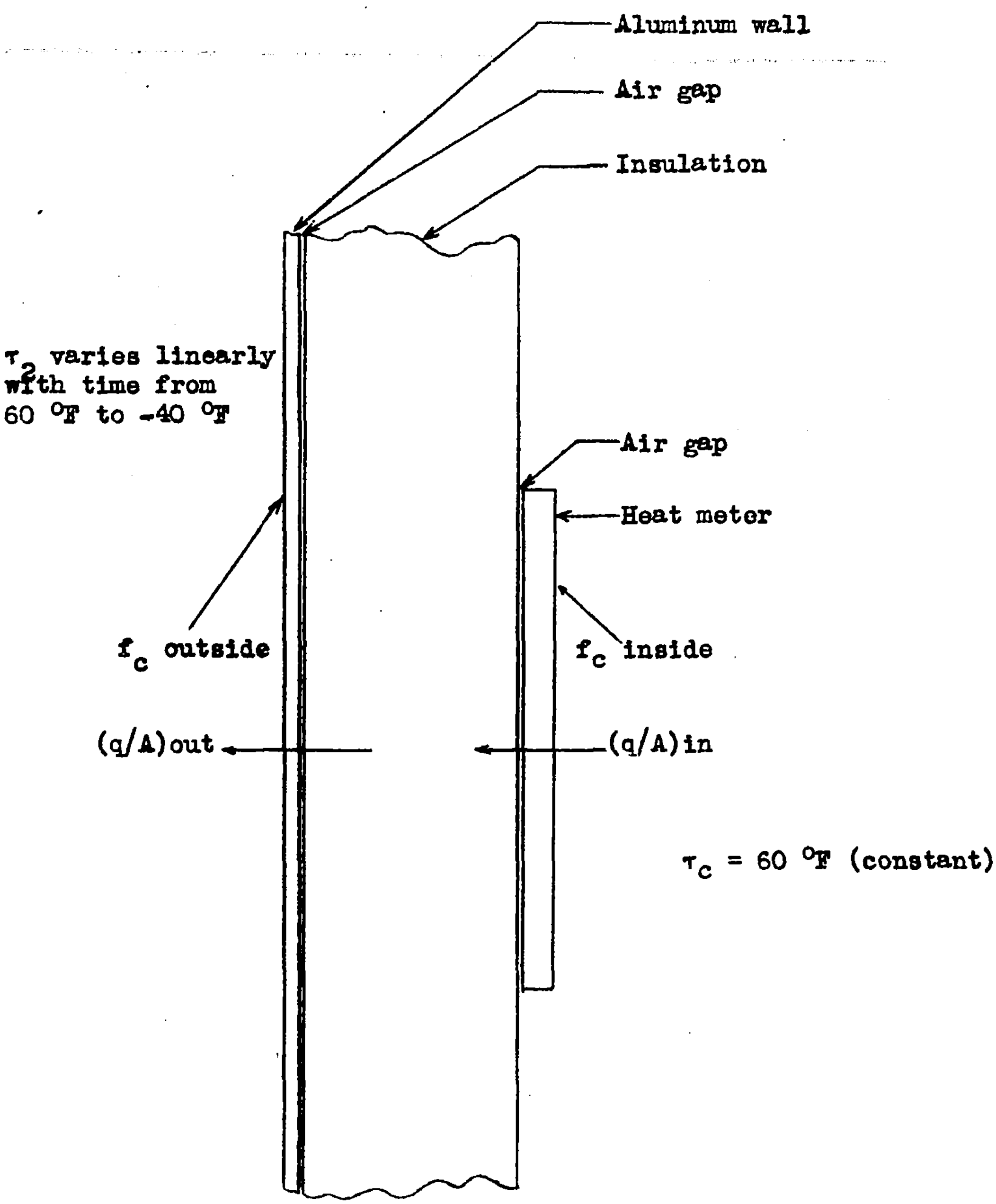


Figure 9.- The thermal circuit of the heat meter placed upon an insulated (composite) cabin wall.

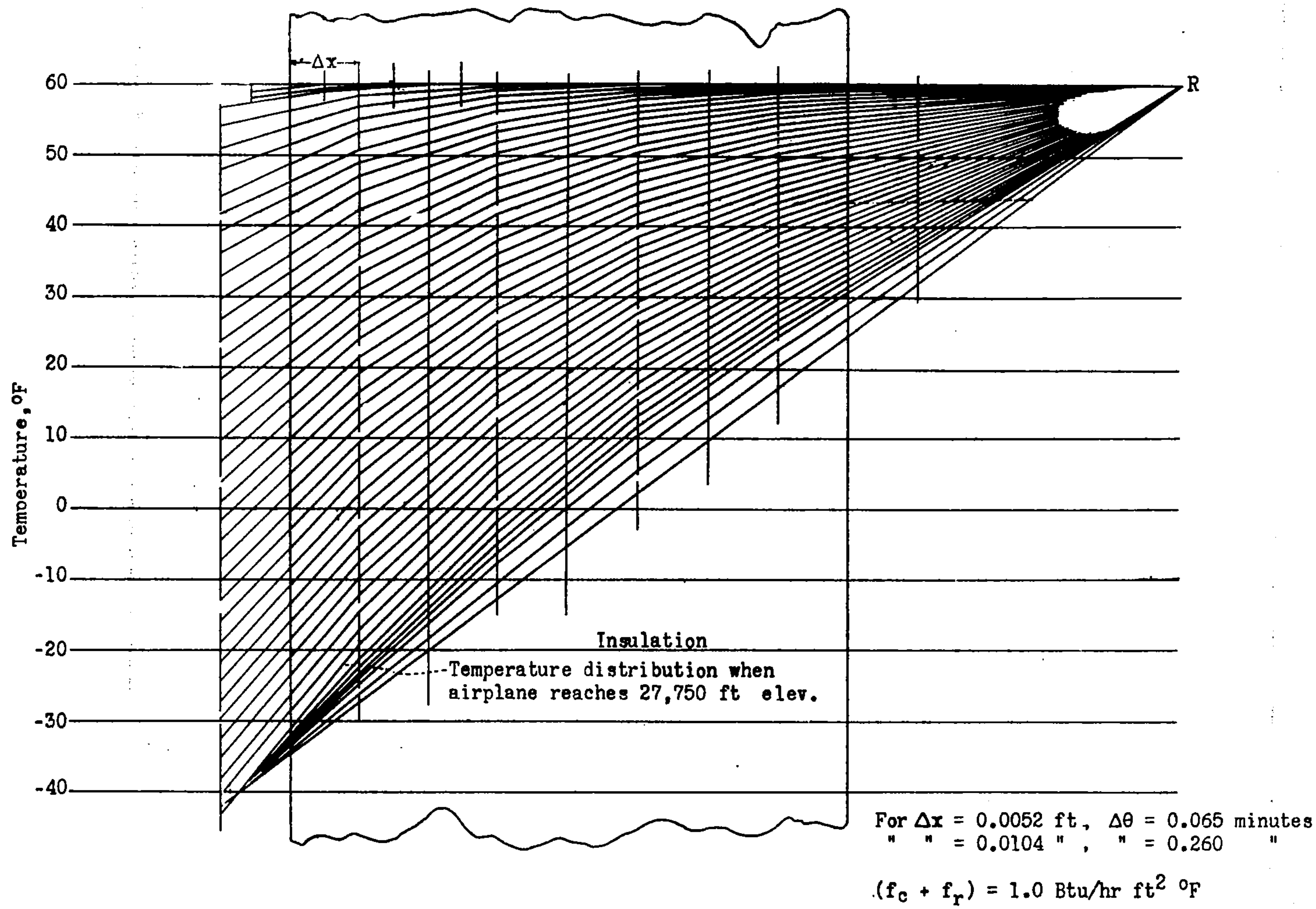


Figure 10.- The Schmidt solution of a heat meter mounted upon the insulated cabin wall of an airplane in flight.

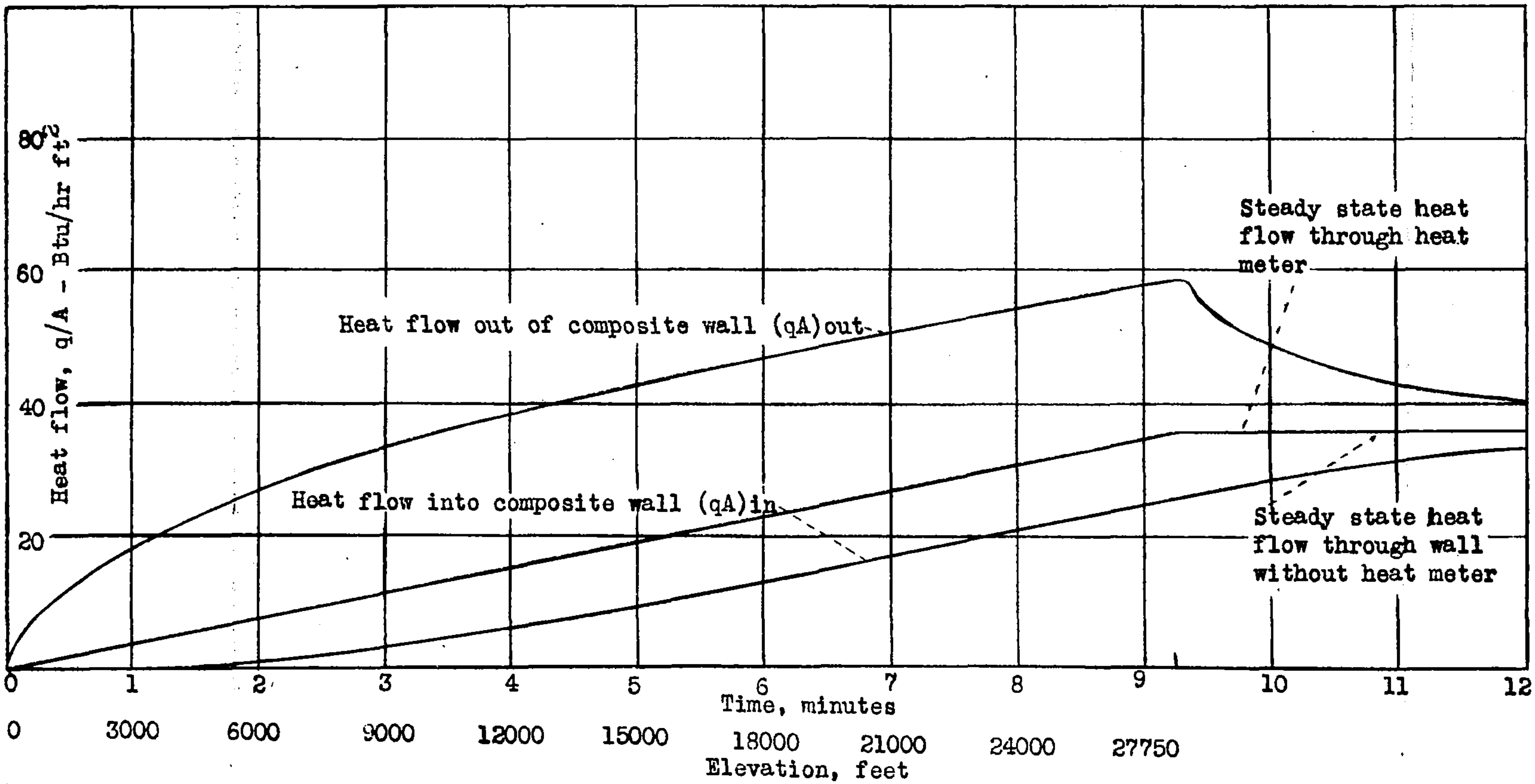


Figure 11.- Schmidt solution of the time-heat flow history of a heat meter mounted on the composite cabin wall of an airplane in flight.

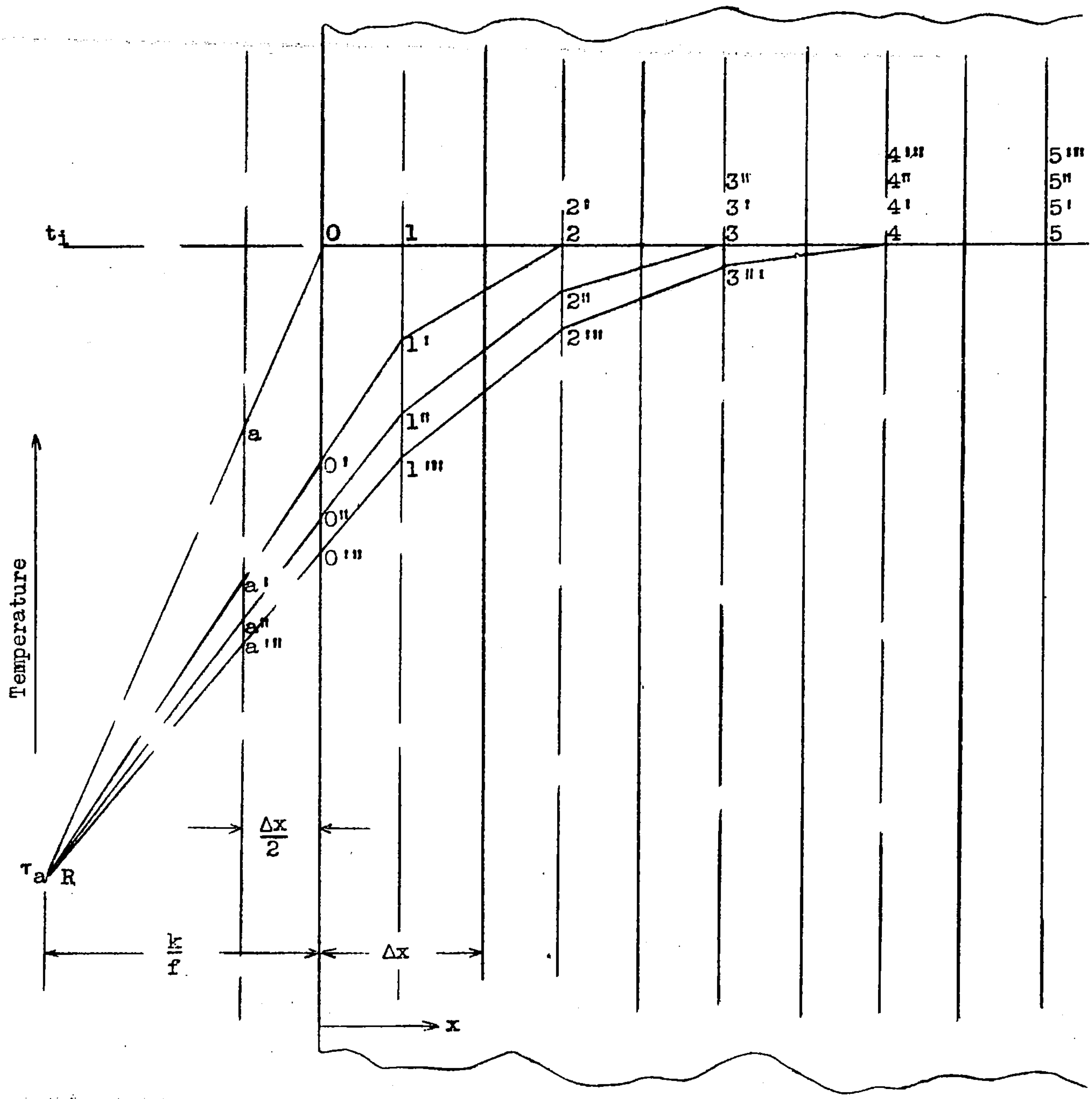


Figure 12.- The sudden cooling of an infinite slab with an air-slab interface resistance.